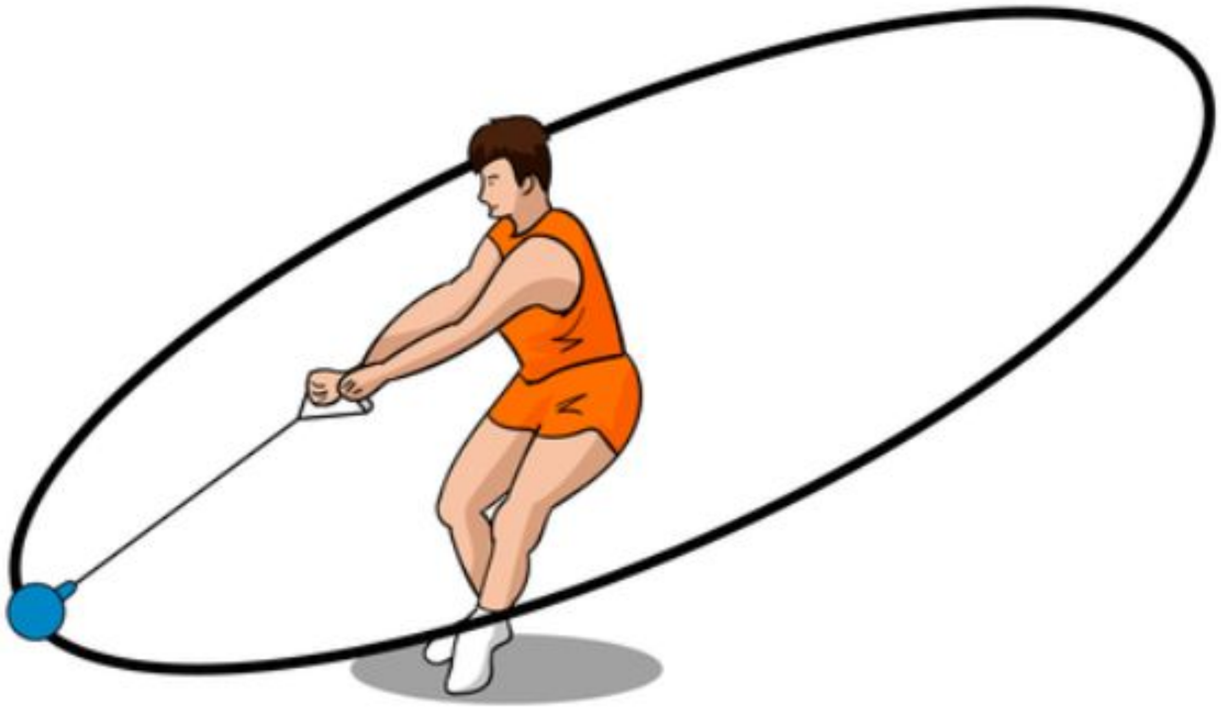
Periodic & Simple Harmonic Motion

Material Covered

Circular Motion

Simple Harmonic Motion

Circular Motion



Specification Points - AQA

3.6.1.1 Circular motion (A-level only)

Content	Opportunities for skills development
Motion in a circular path at constant speed implies there is an acceleration and requires a centripetal force. Magnitude of angular speed $\omega = \frac{v}{r} = 2\pi f$ Radian measure of angle. Direction of angular velocity will not be considered. Centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r$ The derivation of the centripetal acceleration formula will not be examined. Centripetal force $F = \frac{mv^2}{r} = m\omega^2 r$	**MS 0.4** Estimate the acceleration and centripetal force in situations that involve rotation.

Specification Points – OCR A

5.2.1 Kinematics of circular motion

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) the radian as a measure of angle

(b) period and frequency of an object in circular motion

(c) angular velocity ω, $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$

5.2.2 Centripetal force

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) a constant net force perpendicular to the velocity of an object causes it to travel in a circular path

(b) constant speed in a circle; $v = \omega r$

(c) centripetal acceleration; $a = \frac{v^2}{r}$; $a = \omega^2 r$

(d) **(i)** centripetal force; $F = \frac{mv^2}{r}$; $F = m\omega^2 r$

(ii) techniques and procedures used to investigate circular motion using a whirling bung.

Specification Points – OCR B

5.1.2 Out into space

Learning outcomes

(a) *Describe and explain:*

(iv) angular velocity in rad s^{-1}

(v) motion in a horizontal circle and in a circular gravitational orbit.

(c) *Make calculations and estimates involving:*

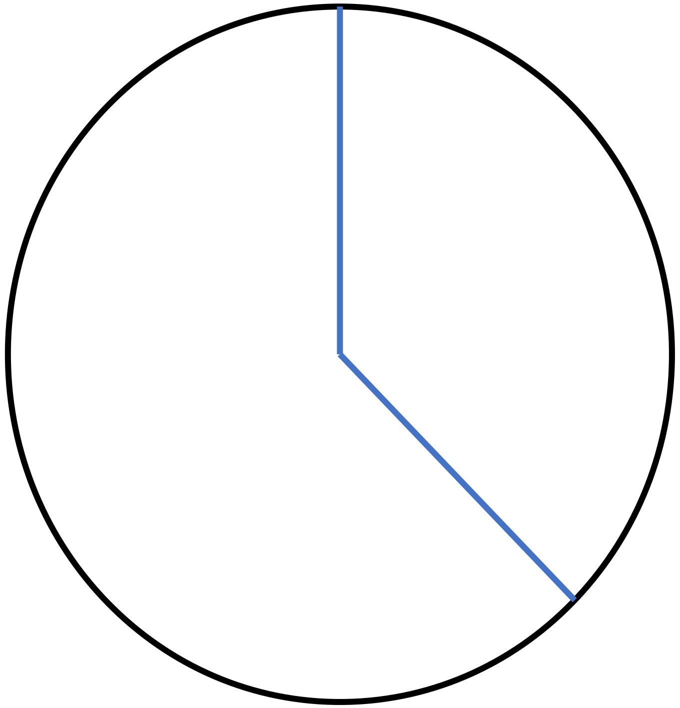
(iii) $a = v^2/r$, $F = mv^2/r = mr\omega^2$

Specification Points - Edexcel

103.	be able to express angular displacement in radians and in degrees, and convert between these units
104.	understand what is meant by *angular velocity* and be able to use the equations $v = \omega r$ and $T = \frac{2\pi}{\omega}$
105.	be able to use vector diagrams to derive the equations for centripetal acceleration $a = \frac{v^2}{r} = r\omega^2$ and understand how to use these equations
106.	understand that a resultant force (centripetal force) is required to produce and maintain circular motion
107.	be able to use the equations for centripetal force $F = ma = \frac{mv^2}{r} = mr\omega^2$

Circular Measures

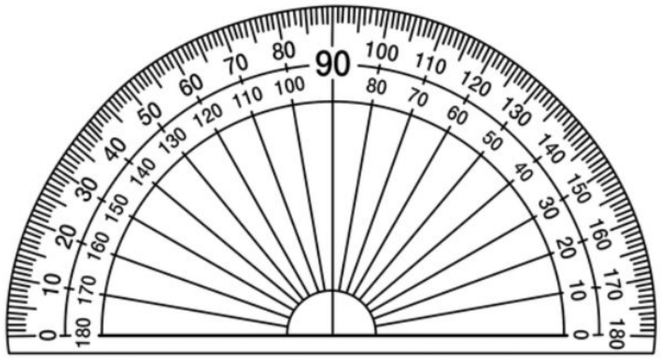
One radian (**rad**) is the **angle** formed by a **circular arc** at the **centre of a circle** when the **arc length** is **equal** to its **radius**.



$$s = r \Rightarrow \theta = 1\ \text{rad}$$

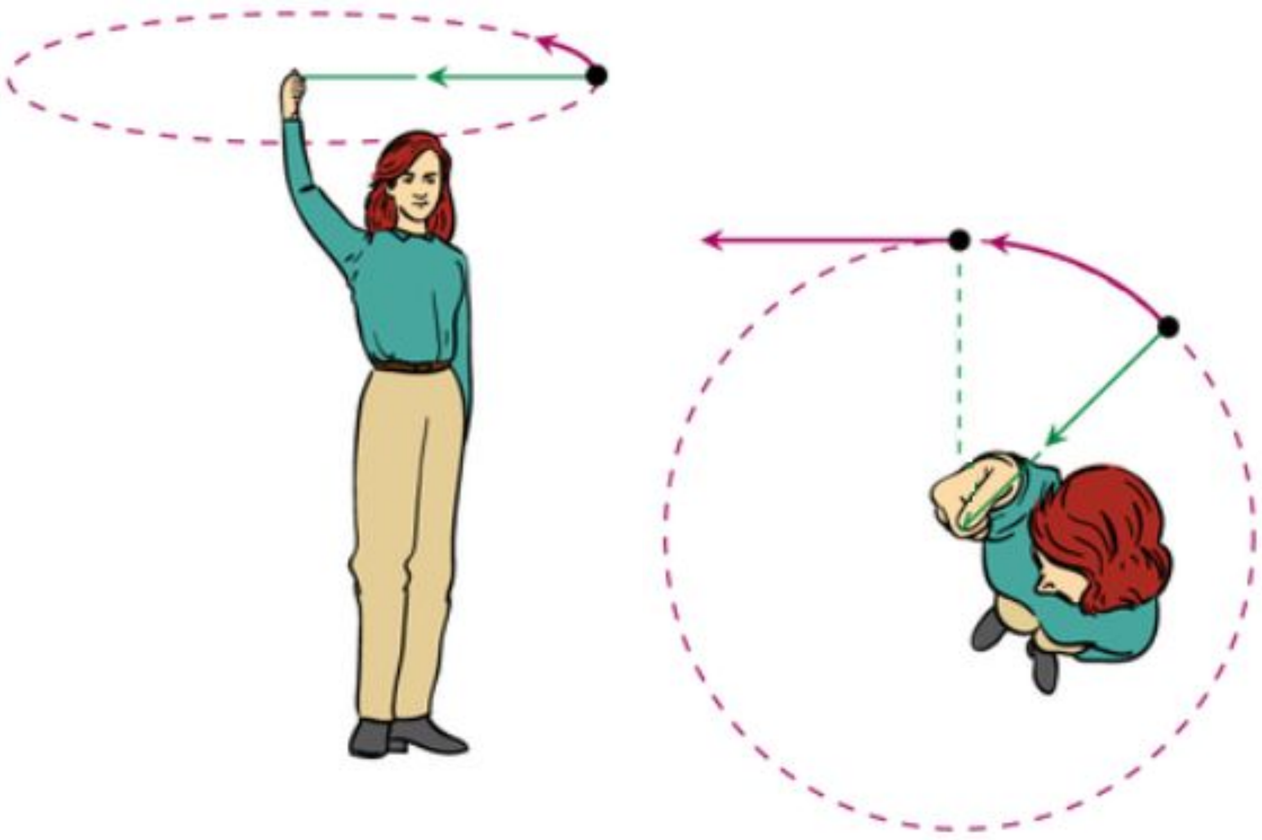
$$s = C \Rightarrow \theta = 2\pi\ \text{rad}$$

$$1\ \text{rad} = \frac{180^\circ}{\pi}$$



We say that an **object** which is **periodically** travelling at a **constant speed** in a **complete circle** is undergoing **uniform circular motion**.

Consider **spinning** a **bob** attached to a **string** above your head in a **circle**.



- The **bob** moves at a **constant speed** but its **velocity** is always **changing** as its **direction** is **changing**.

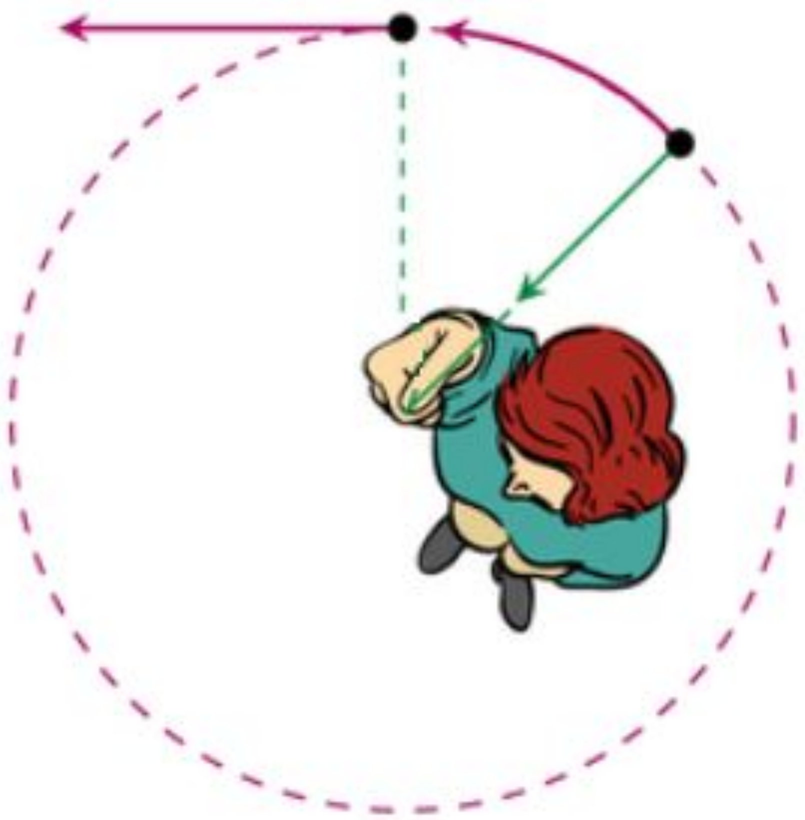
- The **bob** is therefore **accelerating**. We call this **centripetal acceleration**.

- **Angular displacement** $\boldsymbol{\theta}$ is the **angle** swept out in a certain **time**, in **radians**.

- **Period** (T) is **time** to **complete** one **circle**, in **seconds**.

- **Angular speed** $\boldsymbol{\omega}$ is **angular displacement** per **unit time**, in **radians per second**.

- **Instantaneous velocity** $\boldsymbol{v}$ is the **velocity component tangential** to the **circle**, in **metres per second**.



Exemplar Calculation Exam Question

Context: **Circular motion**. May need to determine **period**, **frequency**.

Calculation Question: Mathematical question – we need to show our working.

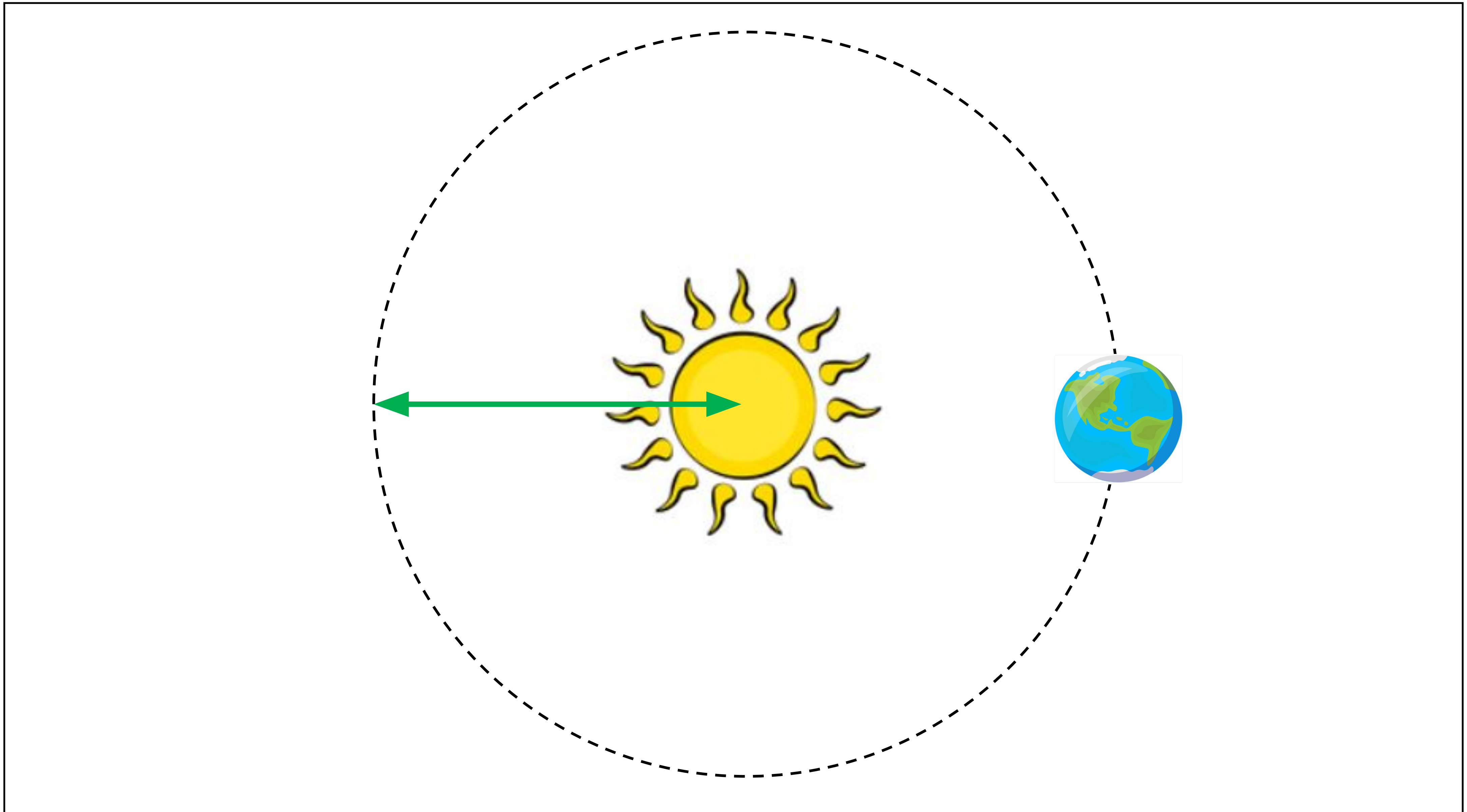
1) The Earth orbits the Sun at a distance of approximately **150** million kilometres. Estimate the angular speed and instantaneous velocity of Earth.

[3 marks]

Answer should be in **units** of **radians per second**.

Answer should be in **units** of **metres per second**.

Indicates that there will be about **3 steps** to our **calculation**. Any less and we may have missed something.



Exemplar Calculation Question Answer

Determine period of **orbit** of **Earth** in **seconds**.

$$T = 1 \text{ year} = 1 \times 365.25 \times 24 \times 60 \times 60 = 31557600 \text{ s}$$ **[1 Mark]**

Write angular speed in terms of **period**.

$$\omega = 2\pi f \quad f = \frac{1}{T} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{31557600}$$

$$= 1.99 \times 10^{-7} \text{rads.s}^{-1}$$

[1 Mark]

Exemplar Calculation Question Answer

Write instantaneous velocity in terms of **period**.

$$v = \omega r \quad = 1.99 \times 10^{-7} \times 1.5 \times 10^{11} \quad = 29900 \text{ ms}^{-1}$$

[1 Mark]

OR

$$v = \frac{2\pi r}{T} \quad = \frac{2\pi \times 1.5 \times 10^{11}}{31557600} = 29900 \text{ ms}^{-1}$$

[1 Mark]

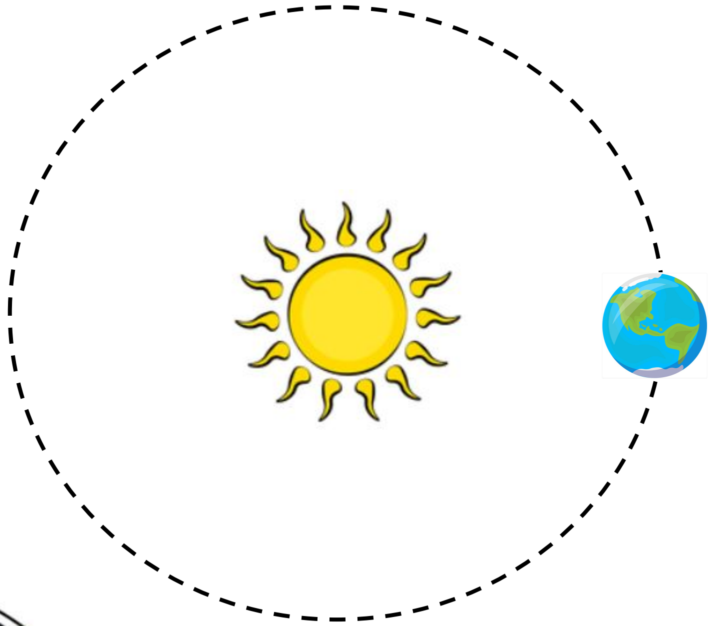
Centripetal Force

For an **object moving** at a **constant speed** v at **radius** r there is an **centripetal acceleration**:

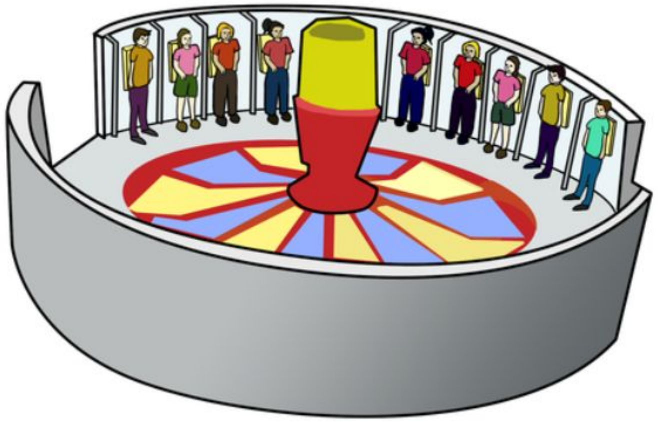
$$a = \frac{v^2}{r}$$

This a has an associated **centripetal force** which always **acts** towards the **centre** of the **circle** (**perpendicular** to **instantaneous velocity**).

Centripetal force is not a **force** which can **act** on its **own** but is always **provided** by some **other force**, **e.g**:



- **Tension** in string.
- **Gravity** (orbits).
- **Friction**.
- **Normal reaction force**.



Exemplar Calculation Exam Question

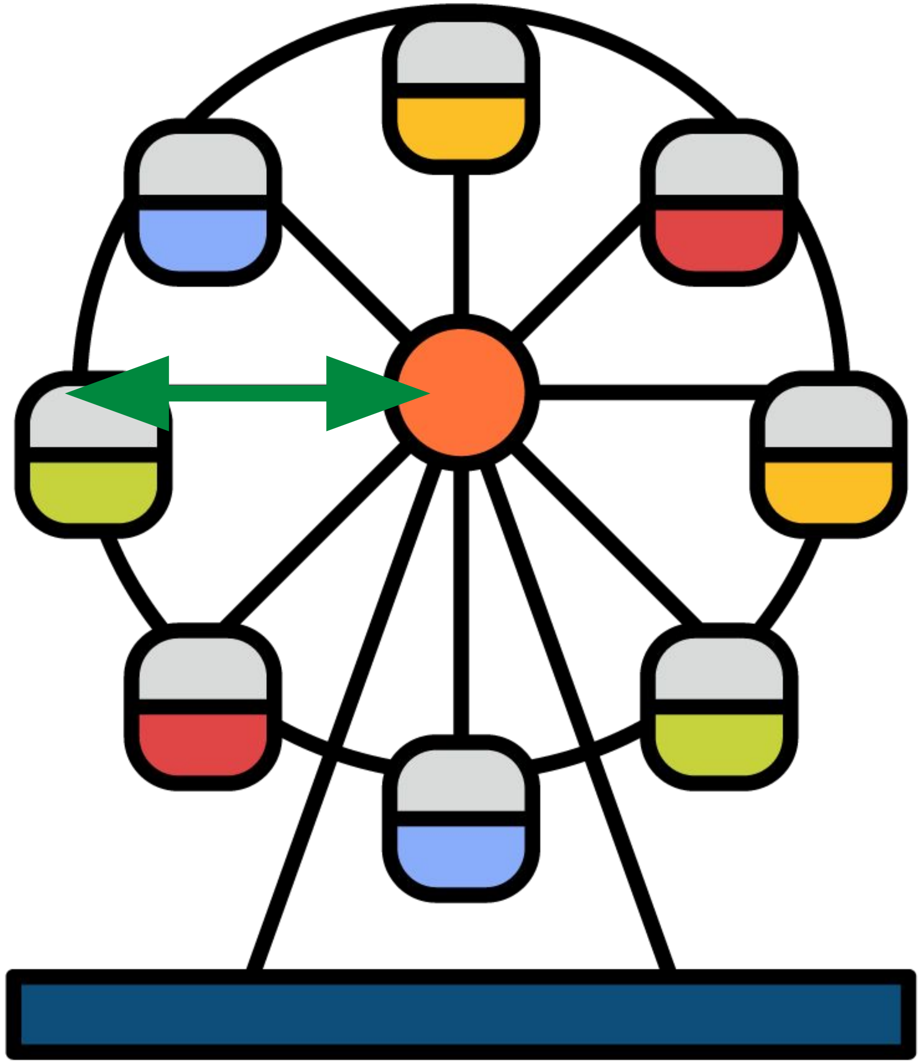
Context: **Circular motion**. May need to determine **period**, **centripetal force**.

Answer should be in **units** of **Newtons.**

1) The London Eye completes a rotation once every 30 minutes while rotating constantly and has a radius of 67.5 meters. Find the magnitude of the force that acts towards the centre of the London eye on a carriage that has a mass of 1000 kg.

[3 marks]

Calculation Question: Mathematical question – we need to show our working.



Exemplar Calculation Question Answer

Recall the formula for centripetal force:

[1 Mark]

Substitute in the equation for velocity:

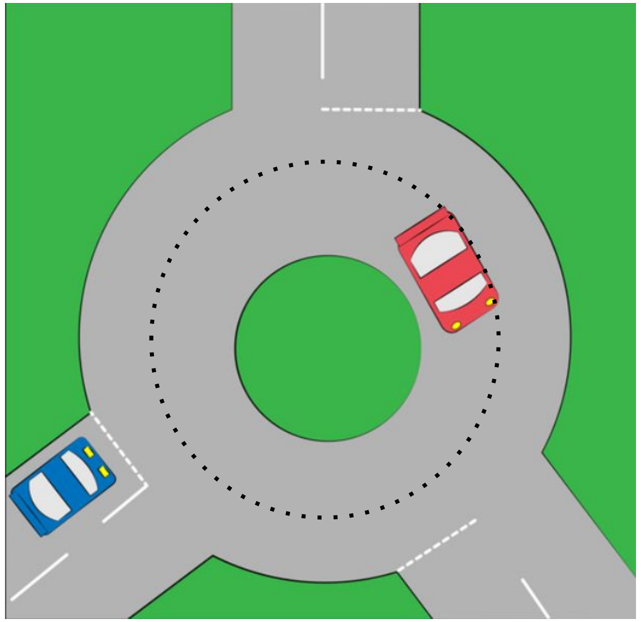
[1 Mark]

Substitute in the values, and give correct units:

[1 Mark]

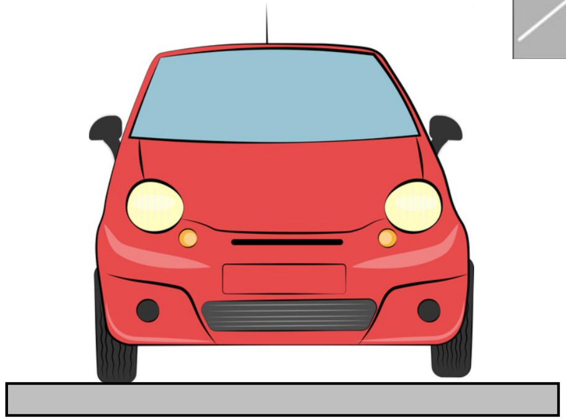
Circular Motion Examples

Consider a **car** driving around a **roundabout**:



- **Friction** between the **tyres** and the **road** provides the **centripetal force**.

- The **equation** for **friction** is: $F_r = \mu R$

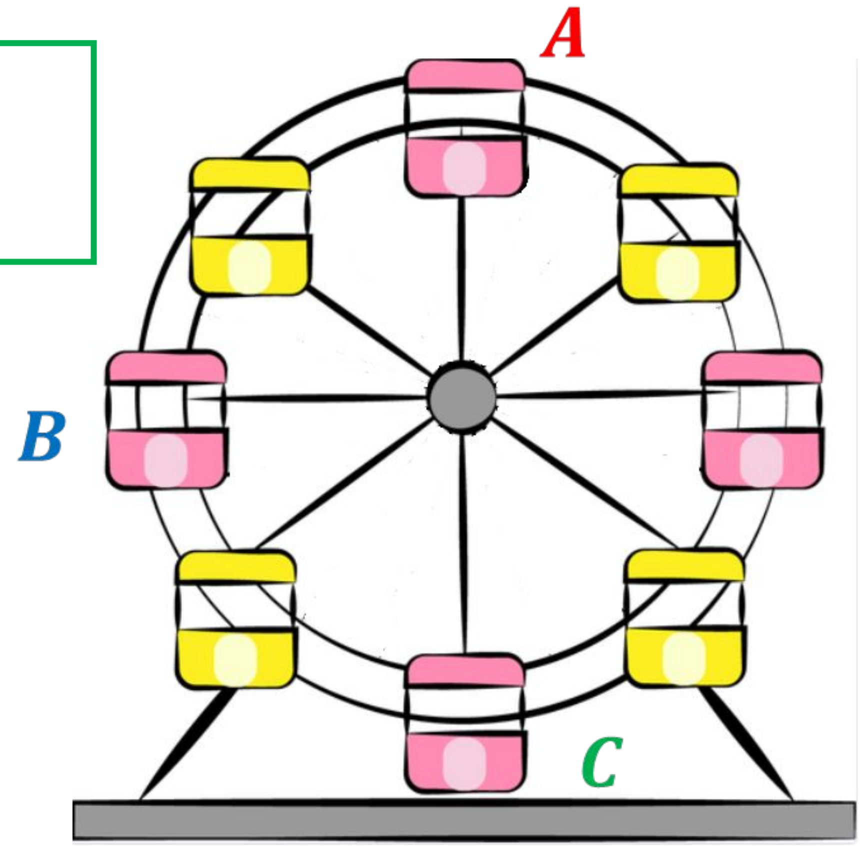


Vertical Circular Motion

Objects can also complete **uniform circular motion** in a **vertical circle**.

The **magnitude** of the **resultant centripetal force** is **constant** but its **components** can **vary**.

- At A: R acts in **same direction** as **weight**.
- At B: R acts **perpendicular** to **weight**.
- At C: R acts **opposite** to **weight**.



Exemplar Explanation Exam Question

Context: **Circular motion**.
Consider **centripetal force**.

Explanation Question: **Bullet-point** format.

1) Inside a washing-machine a sock spins around in a vertical circle stuck to the inside drum. Explain how the normal reaction force acting on the sock varies as it completes the circle.

[3 marks]

Each **point** should mention **normal reaction force acting** on the **sock**.

3 marks means at least **3** points are required.

- Normal reaction force always points perpendicular to surface of drum towards centre of the drum.

[1 Mark]

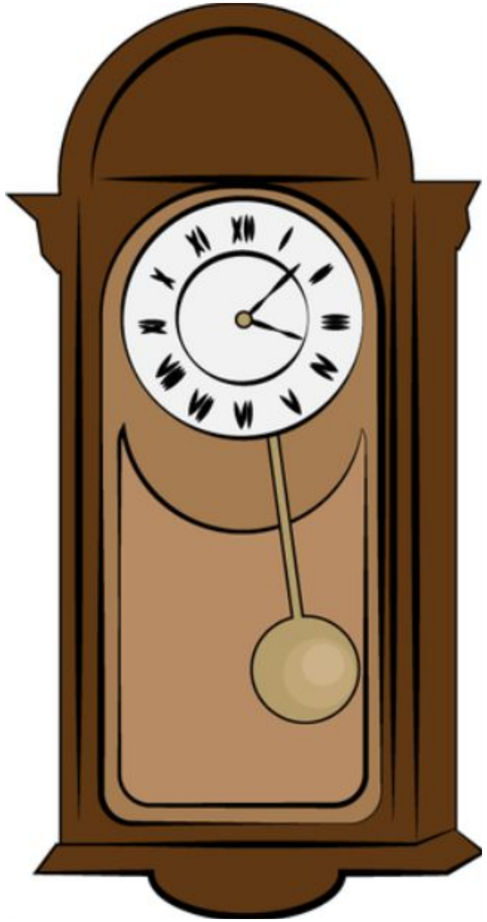
- Magnitude of normal reaction force is maximum at bottom of drum as has to oppose weight and provide centripetal force.

[1 Mark]

- Magnitude of normal reaction force is minimum at top of drum as weight provides at least part of centripetal force.

[1 Mark]

Simple Harmonic Motion



Specification Points - AQA

3.6.1.2 Simple harmonic motion (SHM) (A-level only)

Content	Opportunities for skills development
Analysis of characteristics of simple harmonic motion (SHM). Condition for SHM: $a \propto -x$ Defining equation: $a = -\omega^2 x$ $x = A\cos \omega t$ and $v = \pm \omega\sqrt{(A^2 - x^2)}$ Graphical representations linking the variations of x, v and a with time. Appreciation that the $v - t$ graph is derived from the gradient of the $x - t$ graph and that the $a - t$ graph is derived from the gradient of the $v - t$ graph. Maximum speed = ωA Maximum acceleration = $\omega^2 A$	**AT i, k** Data loggers can be used to produce $s - t$, $v - t$ and $a - t$ graphs for SHM. **MS 3.6, 3.8, 3.9, 3.12** Sketch relationships between x, v, a and t for simple harmonic oscillators.

3.6.1.3 Simple harmonic systems (A-level only)

Content	Opportunities for skills development
Study of mass-spring system: $T = 2\pi\sqrt{\frac{m}{k}}$ Study of simple pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$ Questions may involve other harmonic oscillators (eg liquid in U-tube) but full information will be provided in questions where necessary. Variation of E_k, E_p, and total energy with both displacement and time. Effects of damping on oscillations.	**MS 4.6 / AT b, c** Students should recognise the use of the small-angle approximation in the derivation of the time period for examples of approximate SHM.
Required practical 7: Investigation into simple harmonic motion using a mass–spring system and a simple pendulum.	

3.6.1.4 Forced vibrations and resonance (A-level only)

Content	Opportunities for skills development
Qualitative treatment of free and forced vibrations. Resonance and the effects of damping on the sharpness of resonance. Examples of these effects in mechanical systems and situations involving stationary waves.	**AT g, i, k** Investigation of the factors that determine the resonant frequency of a driven system.

Specification Points – OCR A

5.3.1 Simple harmonic oscillations

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) displacement, amplitude, period, frequency, angular frequency and phase difference

(b) angular frequency ω; $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$

(c) **(i)** simple harmonic motion; defining equation $a = -\omega^2 x$

(ii) techniques and procedures used to determine the period/frequency of simple harmonic oscillations

(d) solutions to the equation $a = -\omega^2 x$ e.g. $x = A\cos\omega t$ or $x = A\sin\omega t$

(e) velocity $v = \pm\omega\sqrt{A^2 - x^2}$ hence $v_{max} = \omega A$

(f) the period of a simple harmonic oscillator is independent of its amplitude (isochronous oscillator)

(g) graphical methods to relate the changes in displacement, velocity and acceleration during simple harmonic motion.

5.3.2 Energy of a simple harmonic oscillator

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) interchange between kinetic and potential energy during simple harmonic motion

(b) energy-displacement graphs for a simple harmonic oscillator

5.3.3 Damping

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) free and forced oscillations

(b) **(i)** the effects of damping on an oscillatory system

(ii) observe forced and damped oscillations for a range of systems

(c) resonance; natural frequency

(d) amplitude-driving frequency graphs for forced oscillators

(e) practical examples of forced oscillations and resonance.

Specification Points – OCR B

5.1.1 Creating models

Learning outcomes

(a) *Describe and explain:*

(v) simple harmonic motion of a mass with a restoring force proportional to displacement such that $\frac{d^2x}{dt^2} = -\frac{k}{m}x$

(vi) simple harmonic motion of a system where $a = -\omega^2 x$, where $\omega = 2\pi f$, and two possible solutions are $x = A\sin(\omega t)$ and $x = A\cos(\omega t)$

(vii) kinetic and potential energy changes in simple harmonic motion

(viii) free and forced vibrations, damping and resonance.

(b) *Make appropriate use of:*

(iii) for oscillating systems:

the terms: simple harmonic motion, period, frequency, free and forced oscillations, resonance, damping

by sketching, plotting from data and interpreting:

(vii) x–t, v–t and a–t graphs of simple harmonic motion including their relative phases

(viii) amplitude of a resonator against driving frequency.

(d) *Demonstrate and apply knowledge and understanding of the following practical activities (HSW4):*

(i) measuring the period/frequency of simple harmonic oscillations for example mass on a spring or simple pendulum and relating this to parameters such as mass and length

(ii) qualitative observations of forced and damped oscillations for a range of systems

(c) *Make calculations and estimates involving:*

(vi) $T = 2\pi\sqrt{\frac{m}{k}}$ with $f = 1/T$ for a mass oscillating on a spring

(vii) $T = 2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum

(viii) $F = kx$; $E = \frac{1}{2}kx^2$

(ix) solving equations of the form $\frac{\Delta^2 x}{\Delta t^2} = -\frac{k}{m}x$ by iterative numerical or graphical methods

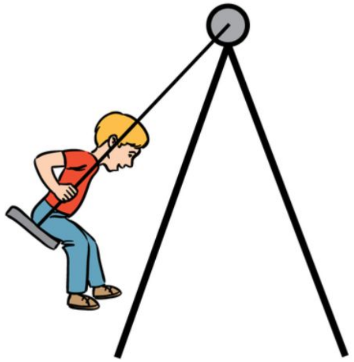
(x) $x = A\sin 2\pi ft$ or $x = A\cos 2\pi ft$

(xi) $E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$.

Specification Points - Edexcel

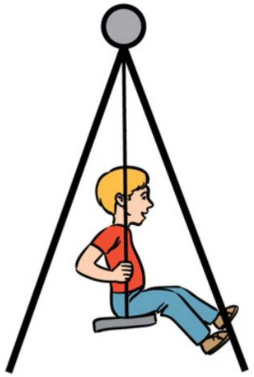
181. understand that the condition for simple harmonic motion is $F = -kx$, and hence understand how to identify situations in which simple harmonic motion will occur	186. understand what is meant by *resonance*
182. be able to use the equations $a = -\omega^2 x$, $x = A\cos\omega t$, $v = -A\omega\sin\omega t$, $a = -A\omega^2\cos\omega t$, and $T = \frac{1}{f} = \frac{2\pi}{\omega}$ and $\omega = 2\pi f$ as applied to a simple harmonic oscillator	187. **CORE PRACTICAL 16: Determine the value of an unknown mass using the resonant frequencies of the oscillation of known masses.**
183. be able to use equations for a simple harmonic oscillator $T = 2\pi\sqrt{\frac{m}{k}}$, and a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$	188. understand how to apply conservation of energy to damped and undamped oscillating systems
184. be able to draw and interpret a displacement–time graph for an object oscillating and know that the gradient at a point gives the velocity at that point	189. understand the distinction between *free* and *forced oscillations*
185. be able to draw and interpret a velocity–time graph for an oscillating object and know that the gradient at a point gives the acceleration at that point	190. understand how the amplitude of a forced oscillation changes at and around the natural frequency of a system and know, qualitatively, how damping affects resonance
	191. understand how damping and the plastic deformation of ductile materials reduce the amplitude of oscillation.

Oscillating Systems



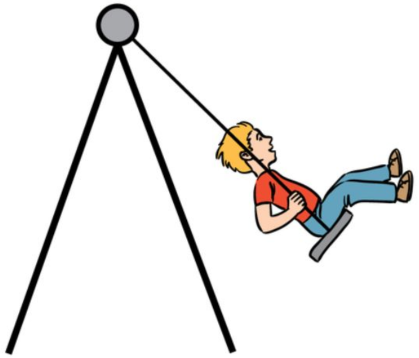
An **oscillating system displaced** from its **equilibrium position** moves **back and forth** around the **equilibrium position**.

Simple Harmonic Motion (**SHM**) is **oscillating motion** of an **object** in which:



- The **magnitude** of **acceleration** (a) is **proportional** to **displacement** from the **equilibrium point** (x).

- The **direction** of **acceleration** is always **opposite** to the **direction** of **displacement**.

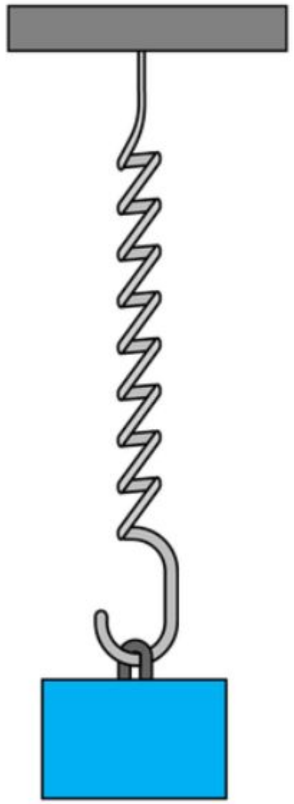


$$a \propto -x$$

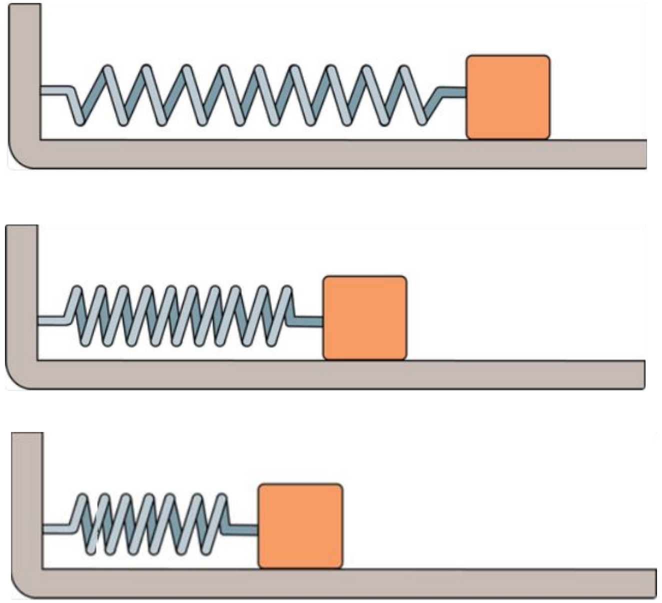
We can **write** this **defining SHM relationship** with a **constant of proportionality**:

$$a = -\omega^2 x$$

- ω is the **angular frequency** of the **motion**.
- The **maximum displacement** from the **equilibrium position** is the **amplitude** A.
- A is **independent** of the **time period** T of the **oscillation**.

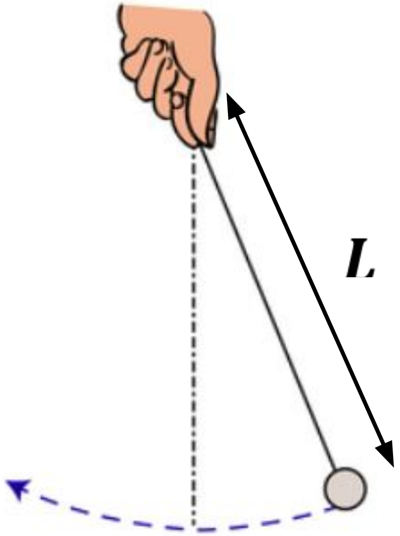


An **example** of a **simple harmonic oscillator** is a **mass**, m, on a **spring** with **spring constant** k.



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Another **example** of a **simple harmonic oscillator** is a **simple pendulum** of **length** L.



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Exemplar Explanation Exam Question

Explanation Question:
Bullet-point format.

Context: **Simple harmonic motion**.
Recall **definitions** and **equations**.

1) Describe and explain how the period and maximum acceleration in the oscillation of a pendulum depends on its initial displacement.

[2 marks]

2 marks means at least **2** points are required.

- Initial displacement determines amplitude A, which period is independent of. **[1 Mark]**

- Maximum acceleration is given by $a_{max} = -\omega^2 A$. Since ω is independent from A, a_{max} is proportional to initial displacement **[1 Mark]**

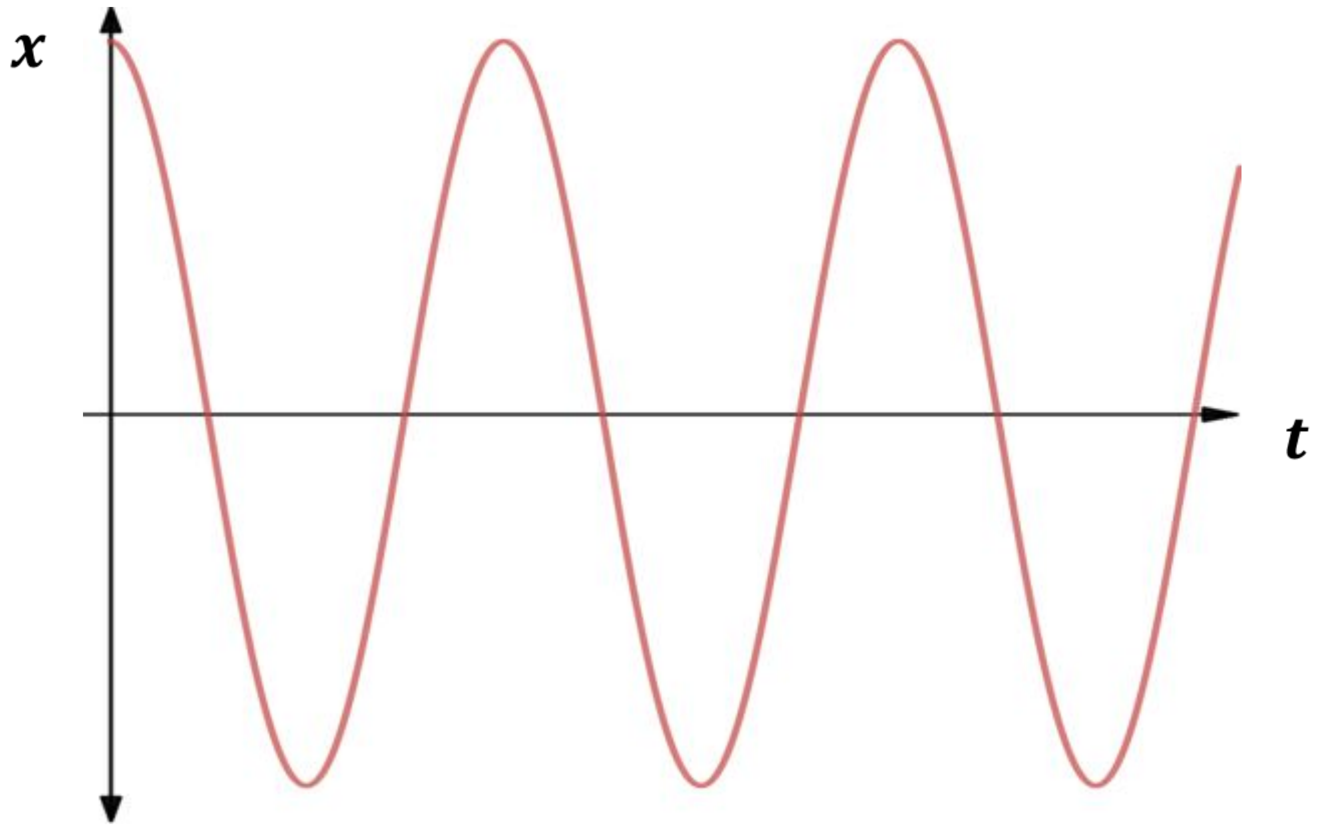
SHM Graphs

We want to find a **solution** for x which **satisfies** the **SHM Equation**: $a = -\omega^2 x$

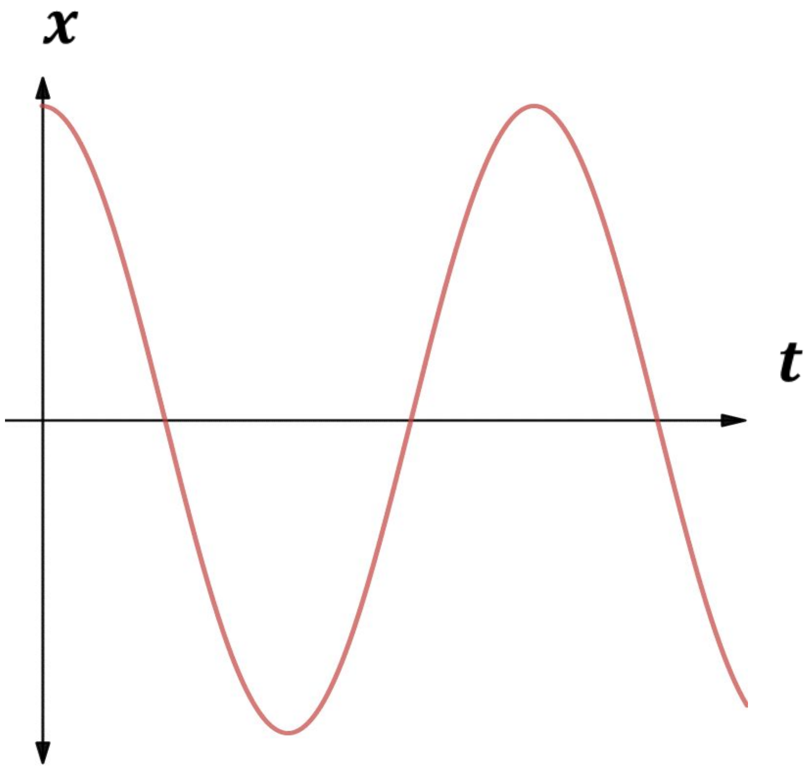
- $x = A\cos(\omega t)$

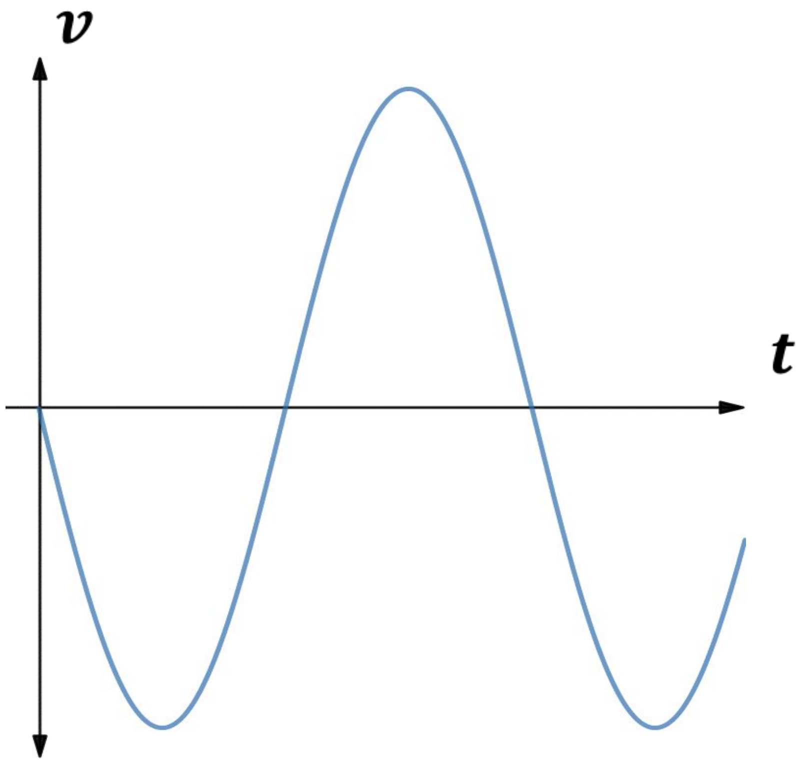
- $v = \frac{dx}{dt} = -A\omega\sin(\omega t)$

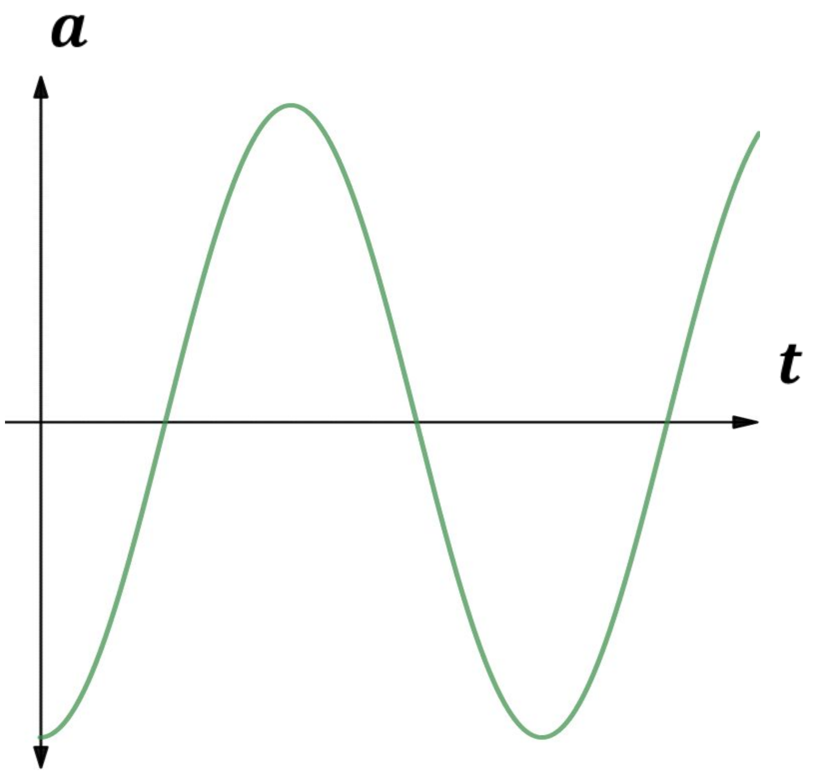
- $a = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t)$



$x = A\cos(\omega t)$ $v = -A\omega\sin(\omega t)$ $a = -A\omega^2\cos(\omega t)$



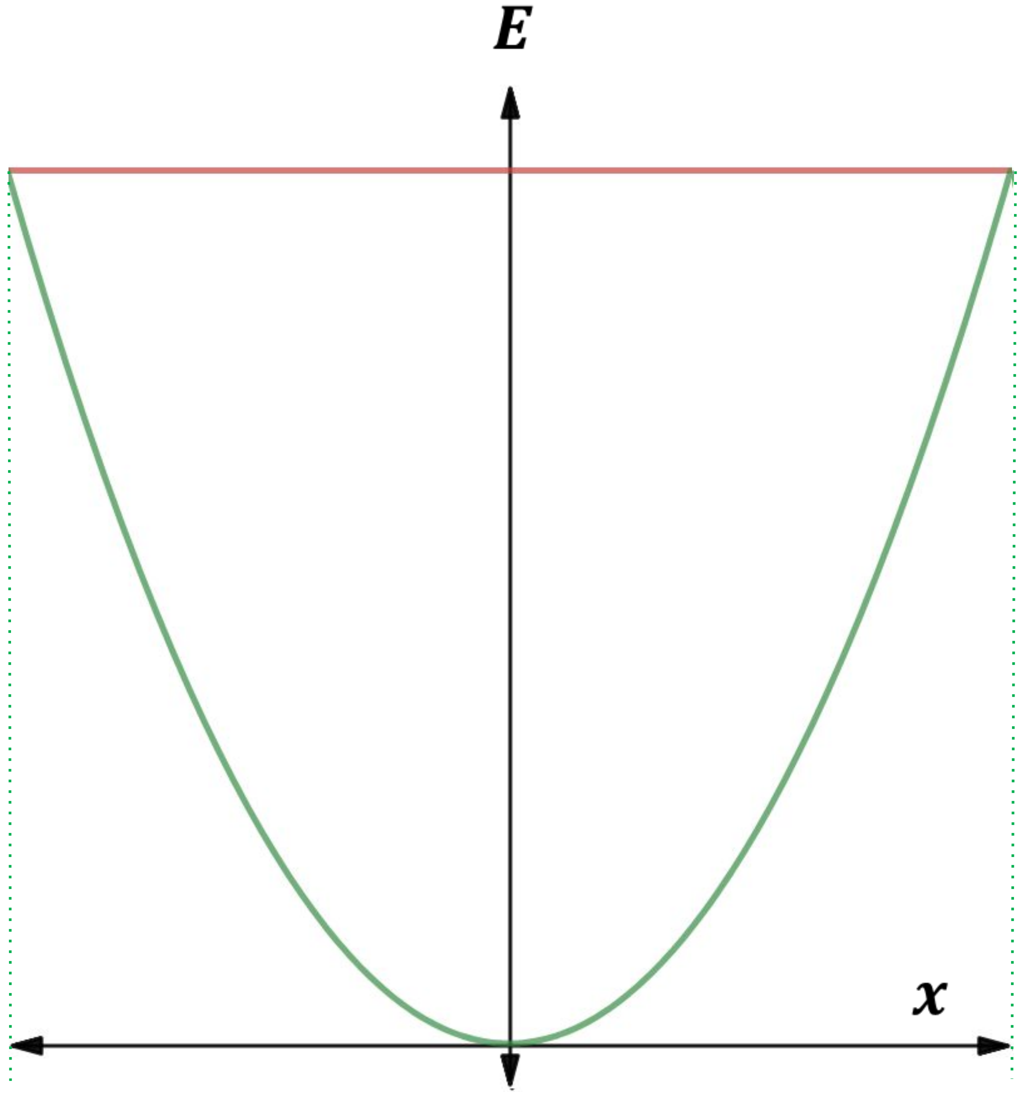




For a **simple harmonic mass-spring system** we can write **equations** for the **energy** of the **system**.

$$E_P = \frac{1}{2}kx^2$$

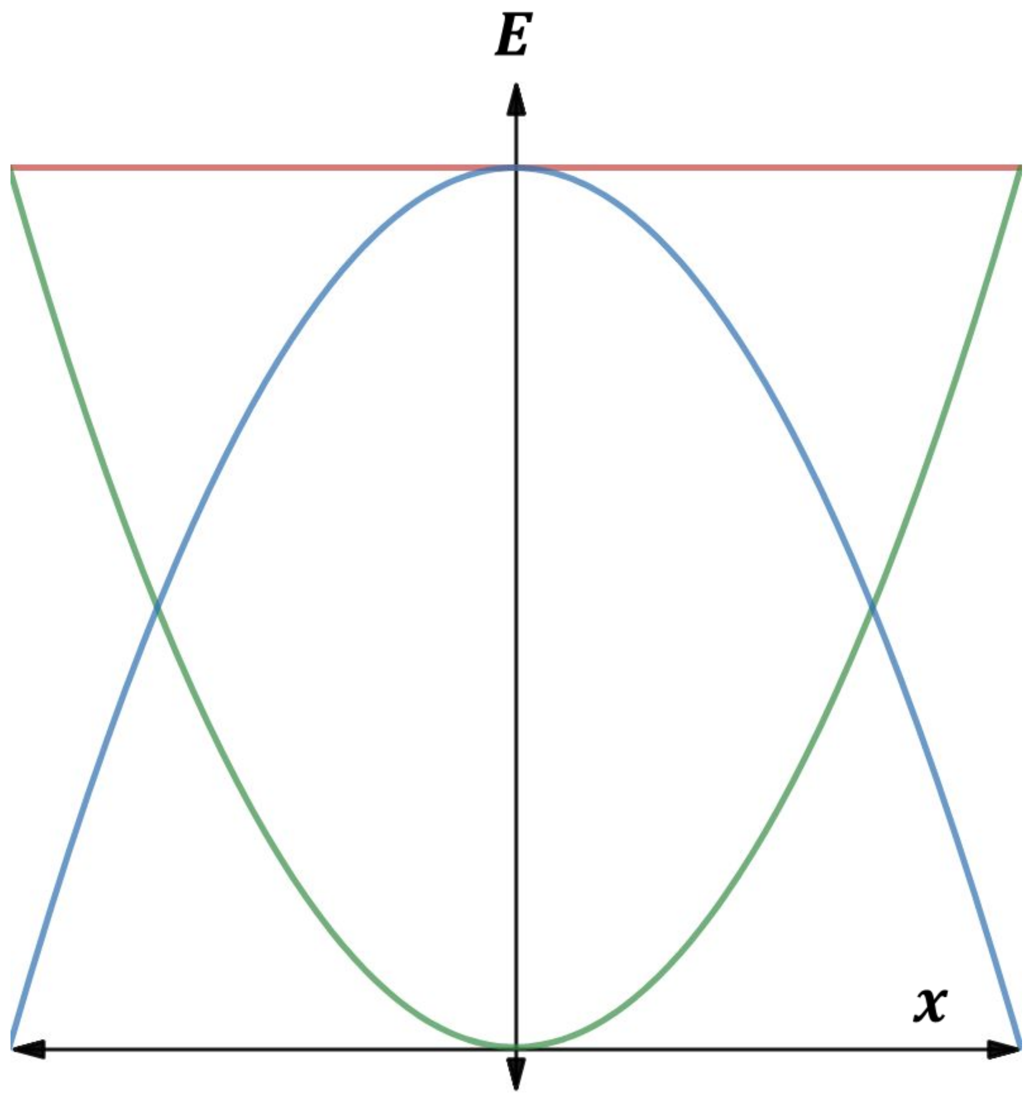
- All energy is stored as E_P at $x = \pm A$ as $v = 0$



Energy is **transferred** between **potential** and **kinetic** but **total energy** is **conserved**.

$$E_K = E_T - E_P$$

$$E_K = \frac{1}{2}mv^2$$



Exemplar Calculation Exam Question

Key values given in **question**.

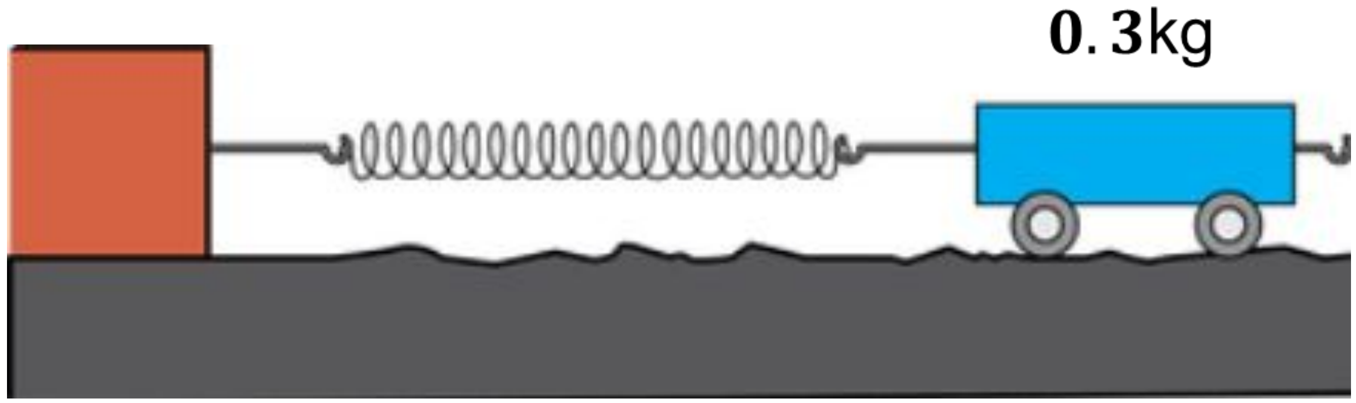
Context: Mass-spring **simple harmonic** system. Recall **equations**.

1) A trolley of mass $\mathbf{0.3\ kg}$ travelling over a frictionless surface is attached to a spring. The trolley is displaced $\mathbf{12\ cm}$ from its equilibrium position such that it undergoes simple harmonic motion with a time period of $\mathbf{0.85\ s}$. Calculate the kinetic energy of the system $\mathbf{2.0\ s}$ after it is released.

[4 marks]

Calculation Question: Mathematical question – we need to show our working.

Indicates that there will be about **4 steps** to our **calculation**.


0.3kg

Exemplar Calculation Question Answer

Calculate angular frequency of **system**

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.85} = 7.391\ldots \text{ rads.s}^{-1}$$

[1 Mark]

Calculate displacement at $t = 2.0$ s

$$x = A\cos(\omega t) = 0.12 \times \cos(7.391\ldots \times 2.0) = -0.07231\ldots \text{ m}$$

[1 Mark]

Exemplar Calculation Question Answer

Determine the **velocity** of the **trolley** at this **displacement**

$$v = \pm\omega\sqrt{A^2 - x^2} \quad = \pm 7.391\ldots\sqrt{0.12^2 - (-0.07231\ldots)^2}$$

$$= \pm 0.7078\ldots\,\mathrm{ms}^{-1}$$

[1 Mark]

Alternatively $v = \pm A\omega\sin(\omega t)$

$$= \pm 0.12 \times 7.391 \times \sin(7.391 \times 2) = \pm 0.7078..\,\mathrm{ms}^{-1}$$

Determine the **kinetic energy** of the **trolley** at this **velocity**

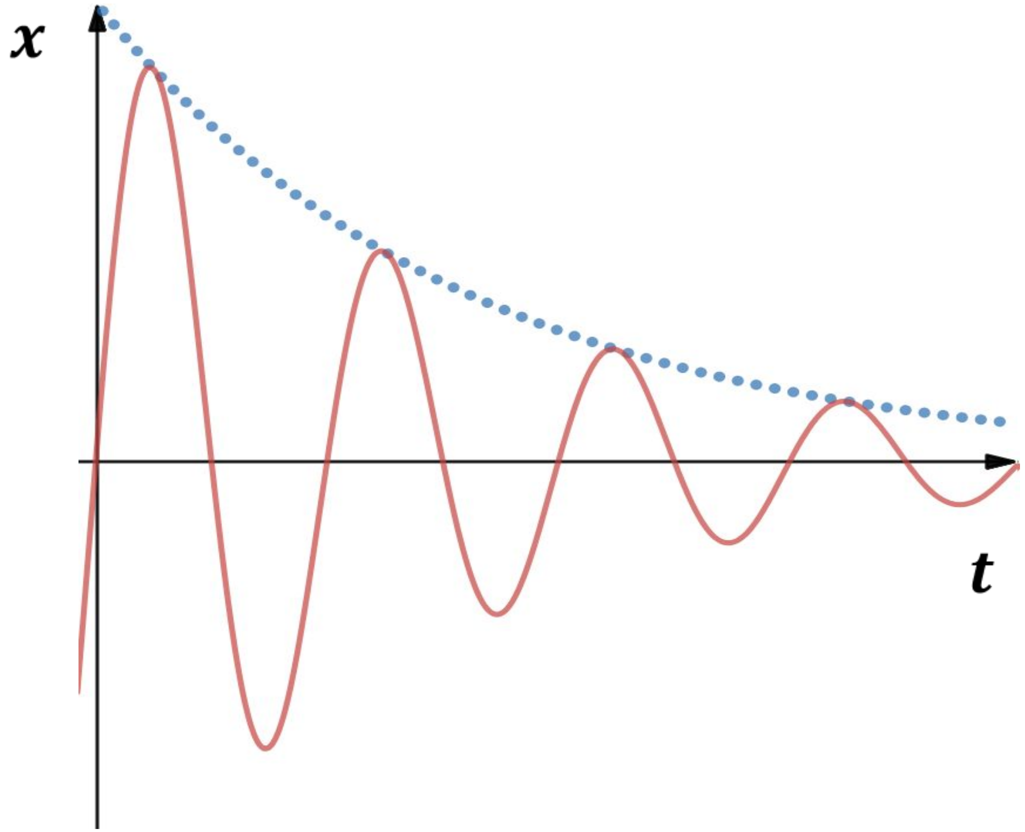
$$E_K = \frac{1}{2}mv^2 \quad = \frac{1}{2} \times 0.3 \times (\pm 0.7078\ldots)^2 = 0.0752\ \mathrm{J}$$

[1 Mark]

Damping

Oscillating systems can **lose** and **dissipate energy** over **time** due to **resistive forces** acting on them, known as **damping**.

- This is **observed** as a **decay** in **amplitude** over **time**.



Damping occurs in **3** different **types**:

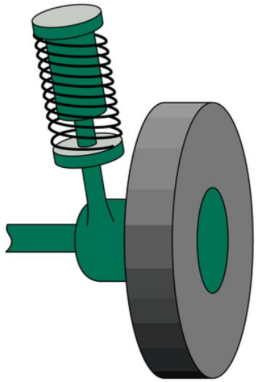
- **Light damping/Underdamping**.
- **Heavy damping/Overdamping**.
- **Critical damping**.

Light damping/Underdamping occurs when the **resistive forces** are **small**.

- **Amplitude** of **oscillation** decays **exponentially**.
- **Time period** remains **constant**.

Heavy damping/Overdamping occurs when the **resistive forces** are **large**.

- **Both amplitude** and **period** of oscillation **decrease**
- Potential for **no oscillations to occur**

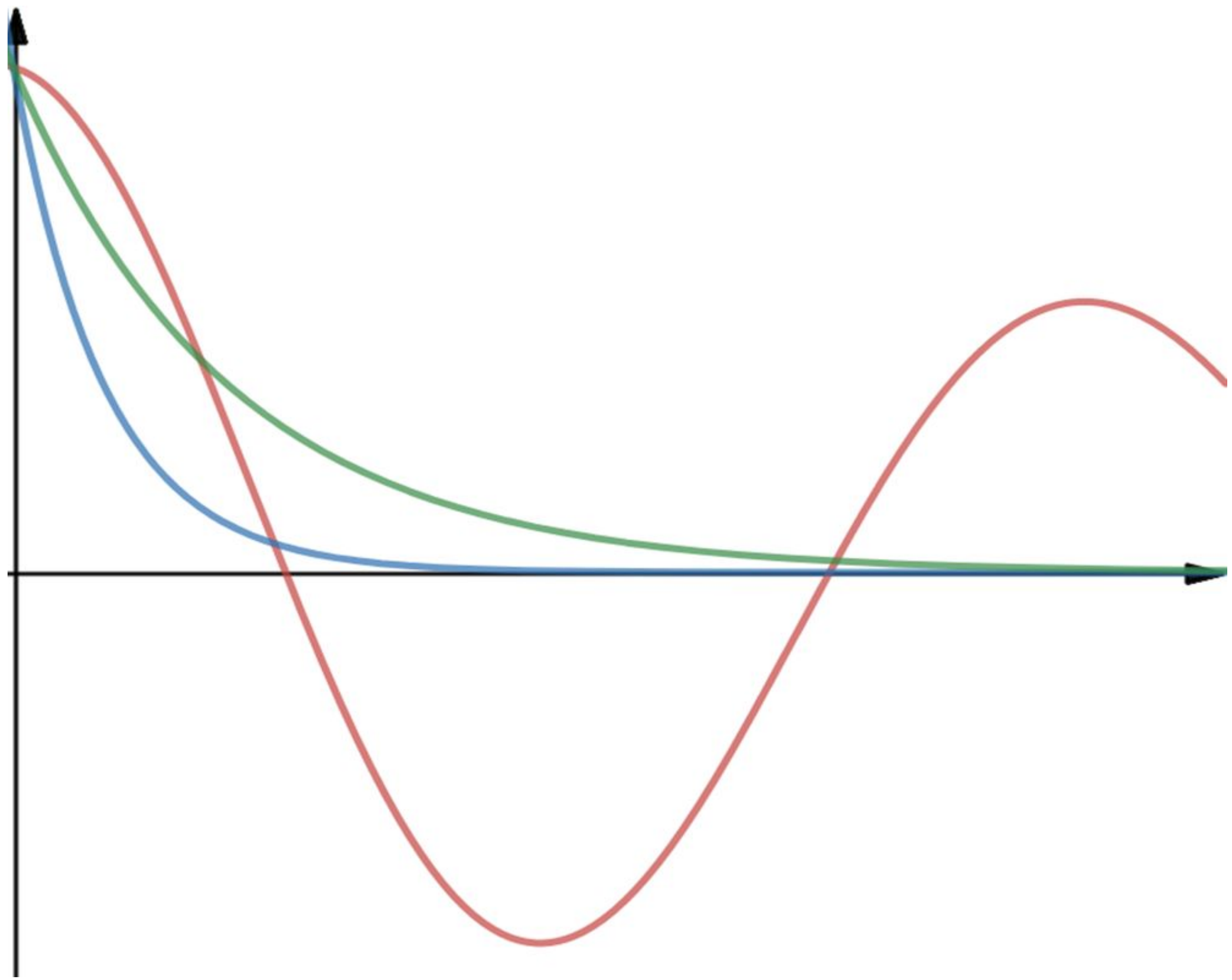


Critical damping occurs when the **resistive forces** are **just right**.

- **System** returns to **equilibrium** in **shortest** possible **time** without **oscillation**.

We can **observe** the **effect** of each **type** of **damping** by **plotting** an object's **displacement** against **time**.

- **Heavy damping/Overdamping**.
- **Critical damping**.
- **Light damping/Underdamping**.



Exemplar Explanation Exam Question

Answer should reference this **type** of **damping**.

Context: **Damped oscillation**. Recall different **regimes**.

1) A grandfather clock mechanism uses a simple pendulum to tell the time. The pendulum is lightly damped. State a possible cause of the light damping and explain why damping does not affect the grandfather clock's accuracy in telling the time.
[2 marks]

Explanation Question: **Bullet-point** format.

2 marks means at least **2** points are required.

- Friction at the pendulum pivot/air resistance removes energy from the system as converted to heat.

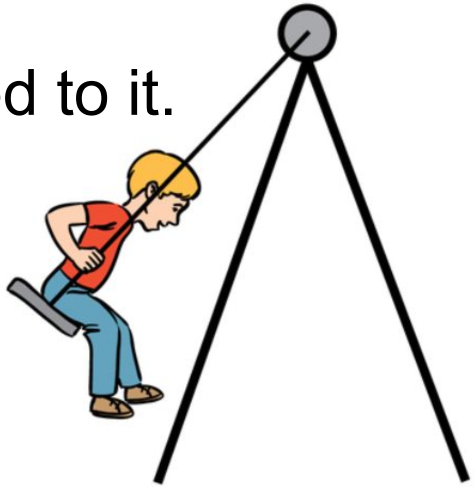
[1 Mark]

- Period of lightly damped system is independent of the amplitude of the oscillation, therefore decrease in amplitude from light damping does not affect period.

[1 Mark]

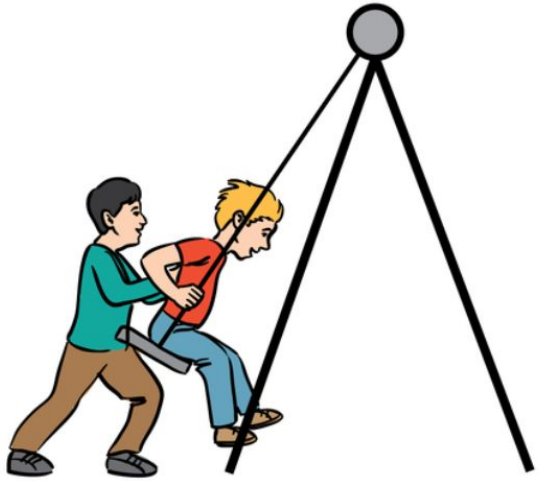
Resonance

A **forced oscillation** has an **external periodic driving force** applied to it.



- The **system oscillates** at the **period** of the **driving force** rather than its **natural frequency**.

When the **driving frequency** is **equal** to the **natural frequency** of the **system, resonance** occurs.

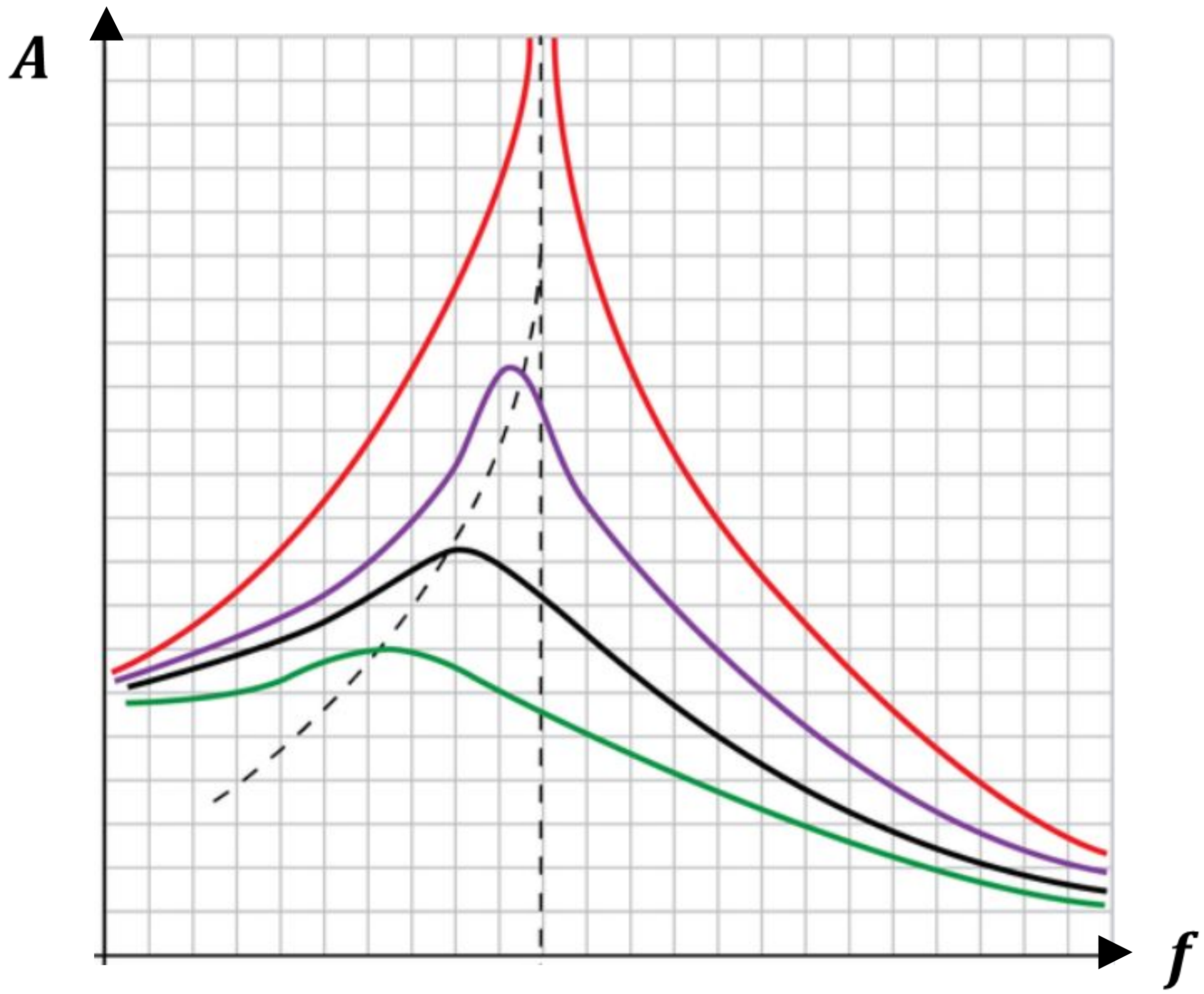


- During **resonance,** the **application** of the **driving force** is **timed** such that it **increases** the **amplitude** of the **oscillation**.

We can **observe** the **effect** of the **driving frequency** on the **amplitude** of **oscillation** by **plotting resonance curves**.

- **Lighter damped systems** have larger **amplitudes** at **resonance**.

- **Lighter damped systems** have a **resonant frequencies** close to their **natural frequency**.



Exemplar Sketch/Plot Exam Question

Context: **Resonance**. **Recall** shape of **graph**.

1) The engine of a car is started and the car sits idle with its handbrake on. The accelerator of the car is revved. Within a certain range of revolutions per minute (rpm) of the engine the chassis of the car can be observed to vibrate. Sketch a graph of amplitude against rpm of the engine for these vibrations and explain the shape of your graph. **[3 marks]**

Sketch/Plot Question:
Make **graph** neat and easy for examiner to **interpret**.

Axes labels.

1 mark for **graph** and **2 marks** for **explaining** its **features**.

Exemplar Sketch/Plot Question Answer

[1 Mark]

- The revolution of the engine causes the car chassis to vibrate at the same frequency.

[1 Mark]

- As the rpm frequency approaches the resonant frequency of the chassis, the chassis vibrations gain energy from driving force and gain amplitude.

[1 Mark]

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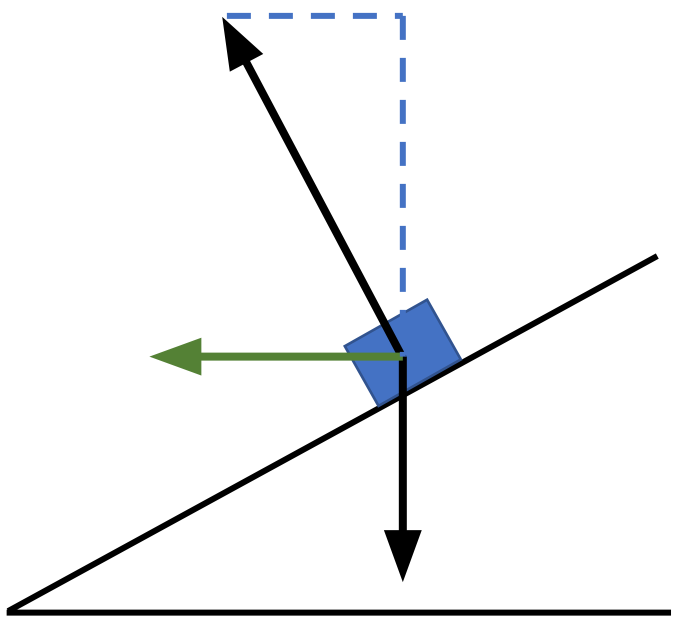
1. Describe the conditions required for resonance. **[2 marks]**

- A periodic driving force must be applied to the system
- with a frequency close to its natural frequency.

2. You are designing a race car track including a corner with radius $\mathbf{125\ m}$. What angle in degrees should the bank make to the horizontal in order to allow cars to drive at $\mathbf{90\ kmh^{-1}}$ around the circular bend without skidding?

[Hint: You may assume that the centripetal force is equal to the horizontal component of the reaction force and the vertical component is equal to the car's weight]

[3 marks]



Equating vertical components of forces:

$$R\cos\theta = mg \quad \rightarrow \quad R = \frac{mg}{\cos\theta}$$

[1 Mark]

Centripetal force provided by horizontal component of reaction force:

$$F_c = \frac{mv^2}{r} = R\sin\theta$$

Substituting in R:

$$\frac{mv^2}{r} = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

[1 Mark]

$$\tan\theta = \frac{v^2}{rg}$$

- Converting from $\mathbf{kmh^{-1}}$ to $\mathbf{ms^{-1}}$

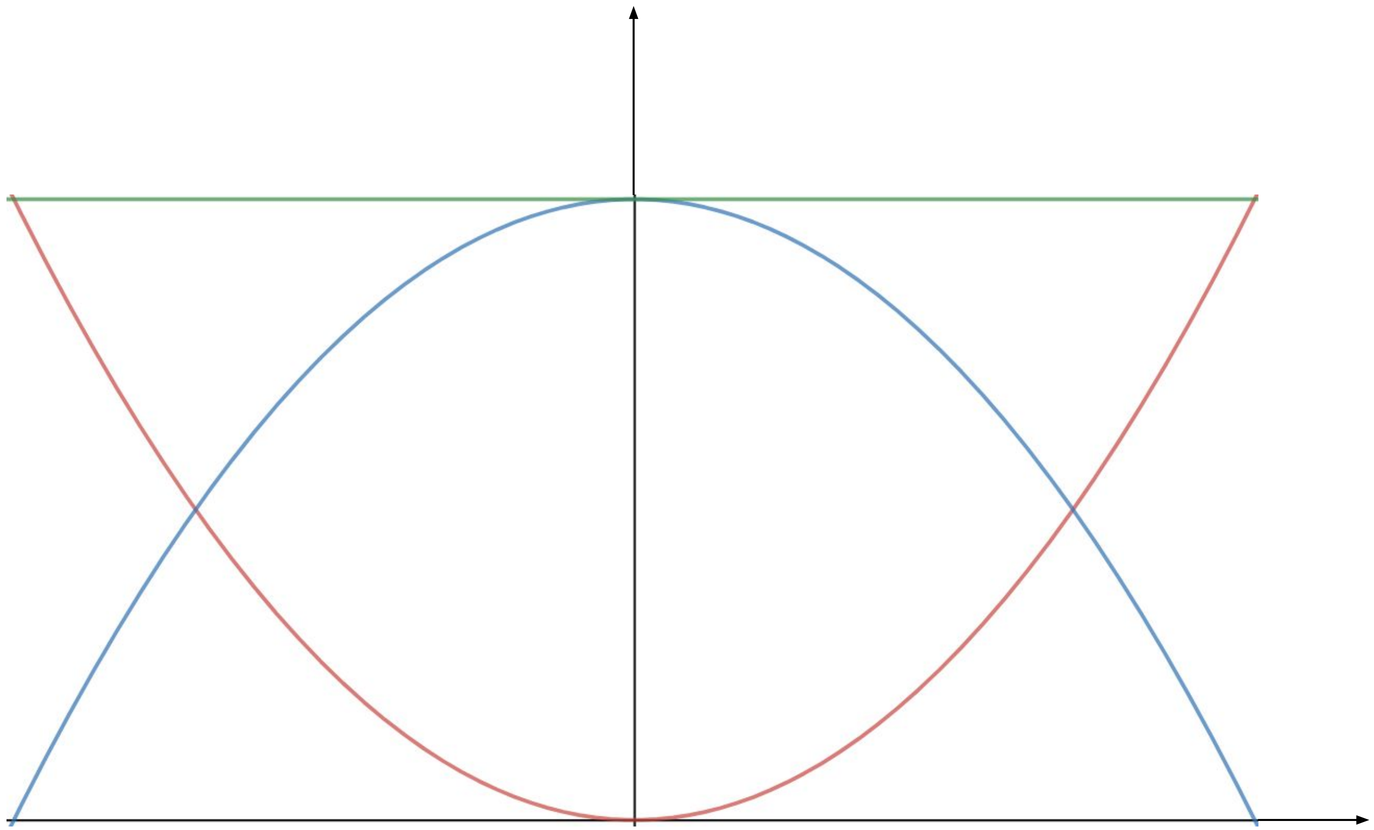
$$\mathbf{90\ kmh^{-1} = \frac{90 \times 10^3}{1 \times 60 \times 60} = 25\ ms^{-1}}$$

- Calculating the angle to the horizontal:

$$\boldsymbol{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{25^2}{125 \times 9.81}\right) = 27.0^\circ}$$ **[1 Mark]**

3. A student carries out an investigation into simple harmonic motion. They hang a mass from a spring with spring constant $\mathbf{25\ Nm^{-1}}$ and displace it by $\mathbf{10\ cm}$, then allow it to oscillate at its natural frequency. Sketch a graph to show the variation with displacement in the kinetic, potential and total energy for the system. Explain the shape of your graph.

[6 marks]



Shape of graph
[1 Mark]

- The potential energy curve is given by $E_p = \frac{1}{2}kx^2$

[1 Mark]

- The total energy of the system is therefore

$$E_T = \frac{1}{2}kA^2 = \frac{1}{2} \times 25 \times 0.1^2 = 0.125\ \text{J}$$

[1 Mark]

- Total energy = potential energy + kinetic energy, so

$$E_k = \frac{1}{2}k\left(A^2 - x^2\right)$$ **[1 Mark]**

4. a) Find the proportionality relationship between the period of a mass on a spring and its spring constant. **[4 marks]**

b) The spring is underdamped. Sketch a graph of it’s displacement against time. **[2 marks]**

a) The frequency for a mass on a spring depends on the spring constant according to: $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ **[1 Mark]**

- Period is inversely proportional to frequency: $T = \frac{1}{f}$ **[1 Mark]**
- Substituting for period and rearranging gives us:

$$\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

[1 Mark]

- Writing out the proportionality: $T \propto \sqrt{\frac{1}{k}}$ (and equivalent forms) **[1 Mark]**

Roughly constant time periods
[1 Mark]

Shape of graph
(Exponential decay)
[1 Mark]