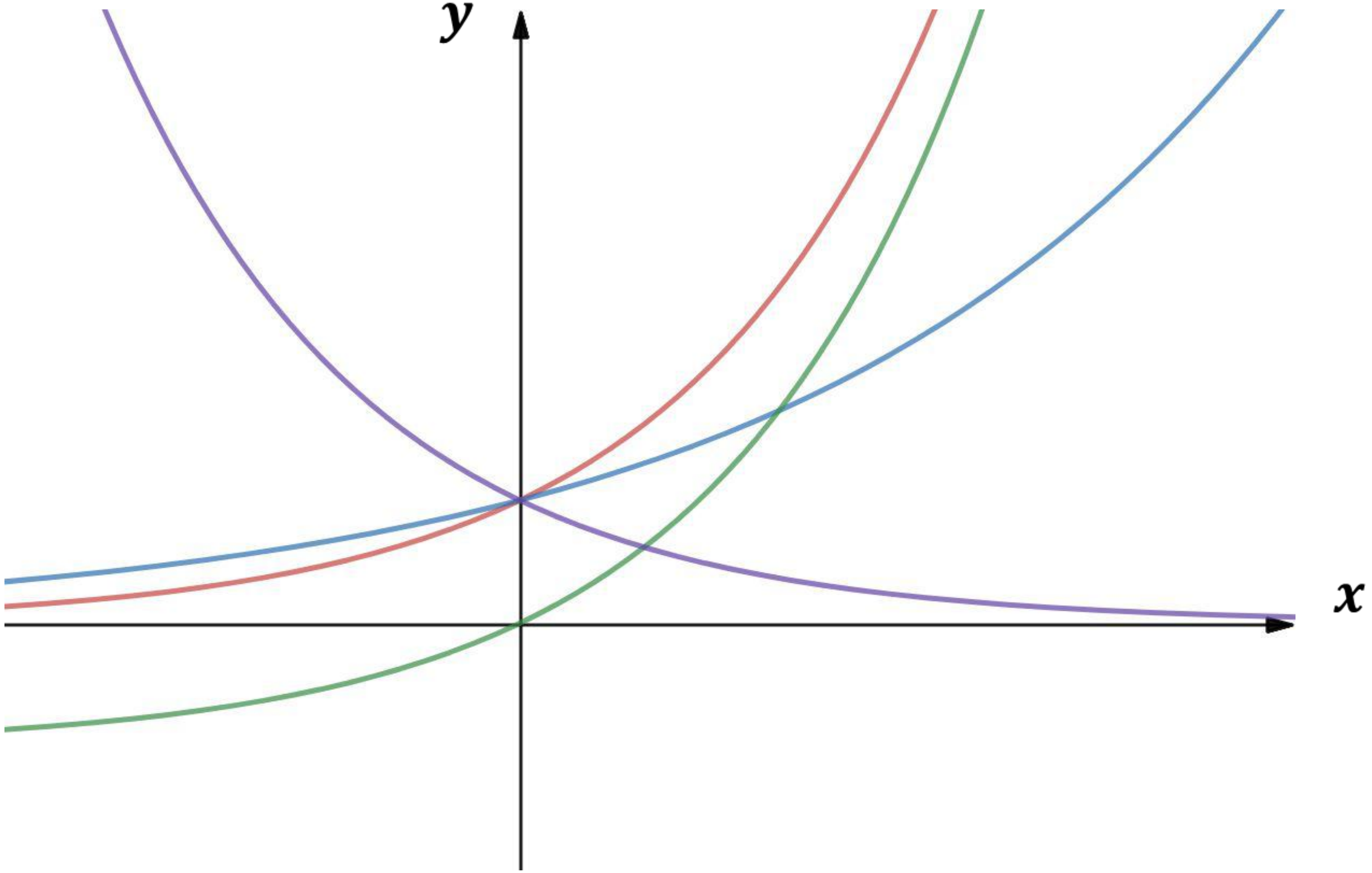
Pure Mathematics: Exponentials & Logarithms



Material Covered

Manipulating Exponentials and Logarithms

1. Exponential and Logarithmic Functions.
2. Exponential and Logarithmic Equations.
3. Exponential and Logarithmic Identities.

Exponential and Logarithmic Graphs

1. Exponential Graphs.
2. Logarithmic Graphs.

Exponential Processes

1. Reduction to Linear Form.
2. Exponential Growth and Decay.

Manipulating Exponentials and Logarithms

$$e^{\ln(x)} = x$$

Specification Points - AQA

	Content
F3	Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$ Know and use the function $\ln x$ and its graph. Know and use $\ln x$ as the inverse function of e^x

	Content
F4	Understand and use the laws of logarithms: $\log_a x + \log_a y \equiv \log_a(xy)$; $\log_a x - \log_a y \equiv \log_a\left(\frac{x}{y}\right)$; $k\log_a x \equiv \log_a x^k$ (including, for example, $k = -1$ and $k = -\frac{1}{2}$)

	Content
F5	Solve equations of the form $a^x = b$

Specification Points – OCR A

OCR Ref.	Subject Content	Stage 1 learners should ...
1.06c	Properties of the logarithm	c) Know and use the definition of $\log_a x$ (for $x > 0$) as the inverse of a^x (for all x),where a is positive. *Learners should be able to convert from index to logarithmic form and vice versa as $a = b^c \Leftrightarrow c = \log_b a$.* *The values $\log_a a = 1$ and $\log_a 1 = 0$ should be known.*
1.06d		d) Know and use the function $\ln x$ and its graph.
1.06e		e) Know and use $\ln x$ as the inverse function of e^x. *e.g. In solving equations involving logarithms or exponentials.* *The values $\ln e = 1$ and $\ln 1 = 0$ should be known.*
1.06f	Laws of logarithms	f) Understand and be able to use the laws of logarithms: 1. $\log_a x + \log_a y = \log_a (xy)$ 2. $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ 3. $k \log_a x = \log_a x^k$ (including, for example, $k = -1$ and $k = -\frac{1}{2}$). *Learners should be able to use these laws in solving equations and simplifying expressions involving logarithms.* *[Change of base is excluded.]*
1.06g	Equations involving exponentials	g) Be able to solve equations of the form $a^x = b$ for $a > 0$ *Includes solving equations which can be reduced to this form such as $2^x = 3^{2x-1}$, either by reduction to the form $a^x = b$ or by taking logarithms of both sides.*

Specification Points – OCR MEI

E2	Be able to convert from an index to a logarithmic form and vice versa.	$x = a^y \Leftrightarrow y = \log_a x$ for $a > 0$ and $x > 0$.
E3	Understand a logarithm as the inverse of the appropriate exponential function and be able to sketch the graphs of exponential and logarithmic functions.	$y = \log_a x \Leftrightarrow a^y = x$ for $a > 0$ and $x > 0$. Includes finding and interpreting asymptotes.
E4	Understand the laws of logarithms and be able to apply them, including to taking logarithms of both sides of an equation.	$\log_a(xy) = \log_a x + \log_a y$ $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ $\log_a(x^k) = k\log_a x$ Including, for example $k = -1$ and $k = -\frac{1}{2}$
E5	Know and use the values of $\log_a a$ and $\log_a 1$.	$\log_a a = 1, \log_a 1 = 0$
E6	Be able to solve an equation of the form $a^x = b$.	Includes solving related inequalities.

Specification Points - Edexcel

6.3	**Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geqslant 0$.** **Know and use the function $\ln x$ and its graph.** **Know and use $\ln x$ as the inverse function of e^x**	$a \neq 1$ **Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.**
6.4	**Understand and use the laws of logarithms:** $\log_a x + \log_a y = \log_a (xy)$ $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ $k \leq \log_a x = \log_a x^k$ **(including, for example,** $k = -1$ and $k = -\frac{1}{2}$)	**Includes $\log_a a = 1$**
6.5	**Solve equations of the form $a^x = b$**	**Students may use the change of base formula. Questions may be of the form, e.g. $2^{3x-1} = 3$**

Exponential and Logarithmic Functions

Exponential functions contain a **variable** x in their **exponent** (also known as their **power** or **index**).

The **function** $f(x) = e^x$ is **unique** as after **differentiation** the **function** is **unchanged**.

$$y = Ae^{kx} \quad \Rightarrow \quad \frac{dy}{dx} = Ake^{kx}$$

Exponential and Logarithmic Functions

The **logarithmic function** $f(x) = \log_a x$ is the **inverse function** of the **exponential function**.

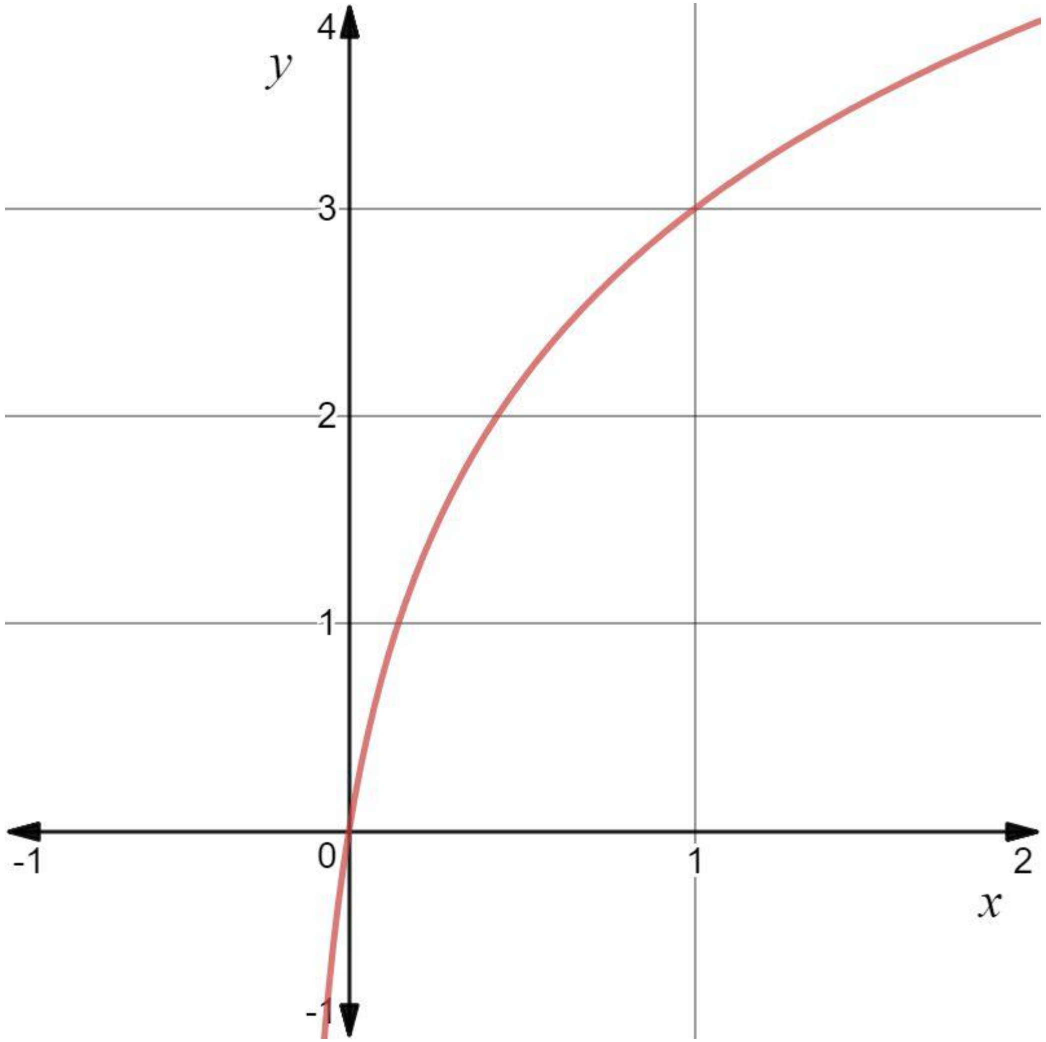
- The **logarithm function** $\log_a x$ asks the question:
  "a to the **power** of what **number** equals x?".
- The **natural logarithm** $f(x) = \ln(x)$ is the **logarithmic function** with **base** e.

Exemplar Exam Question

1) To the right is a segment of the graph $y = \log_2(ax + b)$.

Given that the graph passes through the origin as well as the point $(1, 3)$, deduce the value of a and b.

[3 marks]



Method might not be clear, but **problem won't take long** once we know what to do

3 marks, one for **each value** and one for **method**

Exemplar Exam Question Answer

$y = \log_2(ax + b)$

Use information at origin

When $x = 0$, we know that $y = 0$.

Substitute $x = 0, y = 0$ into given equation.

$0 = \log_2(a \times 0 + b)$

$0 = \log_2 b$

Exemplar Exam Question Answer

$$y = \log_2(ax + b)$$

Deduce value of b

$$0 = \log_2 b$$

Recall when a log is equal to zero:

$$\log_2 b = 0 \Rightarrow b = 2^0$$

Therefore $b = 1$

[1 Mark]

Exemplar Exam Question Answer

$y = \log_2(ax + 1)$

Use information at $(1, 3)$

When $x = 1$, we know that $y = 3$

Substitute $x = 1, y = 3$ into given equation.

$3 = \log_2(a \times 1 + 1)$

This can be used to deduce value of a.

Exemplar Exam Question Answer

$y = \log_2(ax + b)$

Deduce value of a

$3 = \log_2(a + 1)$ **[1 Mark]**

Recall definition of a logarithm: $\log_a x = n \Rightarrow a^n = x$

So $\log_2(a + 1) = 3 \Rightarrow a + 1 = 2^3 = 8$

Therefore $a = 7$ **[1 Mark]**

Exponential and Logarithmic Equations

We can **manipulate expressions** containing **logarithms** with the **Laws of Logarithms**:

- **Multiplication Rule:**

$$\log_a x + \log_a y = \log_a (xy)$$

- **Division Rule:**

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

- **Power Rule:**

$$\log_a x^n = n \log_a x$$

Exponential and Logarithmic Equations

We can use the **Laws of Logarithms** to **solve** for x **equations** of the **form**: $a^{px+q} = b$.

1. Take **logarithms** of both sides.

2. Apply the **power rule**.

3. Make x the **subject**.

4. Find the **answer** using a **calculator**.

Exemplar Exam Question

x is an **index** – we'll need to use **Laws of Logarithms** to bring it down

1) Find x where $e^{2x+1} = 2$. Give your answer to 3 significant figures

[3 marks]

We’ll need a **calculator** at the end

Expect around **3 steps** to our method

Exemplar Exam Question Answer

$$e^{2x+1} = 2$$

Take logarithms of both sides - can use base e

$$\ln\left(e^{2x+1}\right) = \ln 2$$

[1 Mark]

Tidy up left hand side with log laws

Using Multiplication Rule: $\log_a x^n = n \log_a x$

$$(2x + 1)\ln(e) = \ln 2$$

Exemplar Exam Question Answer

$(2x+1)\ln(e) = \ln 2$

Using Identity Rule: $\log_a a = 1$

$2x + 1 = \ln 2$

[1 Mark]

Make x the subject of the equation and solve

$$x = \frac{1}{2}(\ln 2 - 1) = -0.1534264\ldots$$

$$= -0.153 \text{ (to 3 s.f.)}$$

[1 Mark]

Exponential and Logarithmic Identities

There are several other **relations** involving **exponentials** and **logarithms** which can help to **simplify expressions**:

- $\log_a a = 1$
- $\log_a 1 = 0$
- $\log_a \frac{1}{x} = -\log_a x$
- $a^{\log_a x} = x$

Exemplar Exam Question

We need to **rearrange** the equation to find x. The problem seems to **resemble a quadratic equation**

1) Solve

First part seems **much simpler**

(i) $\ln y = \ln \frac{1}{y} + 1$

(ii) $3^{2x} - 12(3^x) + 32 = 0$

Your values for both y and x should ***not*** be written in terms of logarithms

[5 marks]

This instruction tells us a lot:

- We'll be using **logarithms** in this equation.
- We'll be finding multiple solutions in both parts, could involve quadratics.

5 marks in total, most marks probably for second part.

Exemplar Exam Question Answer

$$\ln y = \ln \frac{1}{y} + 1$$

Simplify to a single logarithm

$$\ln y - \ln \frac{1}{y} = 1$$

Use $\log \frac{1}{a} = -\log a$ and $\log a + \log b = \log ab$

$$\ln y^2 = 1$$

$$\Rightarrow y^2 = e^1$$

$$y = \sqrt{e}$$

[1 Mark]

Exemplar Exam Question Answer

$3^{2x} - 12(3^x) + 32 = 0$

Use index rules to rearrange equation

Using $a^{nx} = (a^x)^n$

$3^{2x} - 12(3^x) + 32 = 0 \quad \rightarrow \quad (3^x)^2 - 12(3^x) + 32 = 0$ **[1 Mark]**

Treat new equation as a quadratic

Let $y = 3^x$

Then $y^2 - 12y + 32 = 0$ **[1 Mark]**

Exemplar Exam Question Answer

Solve equation in terms of y

Factorise: $y^2 - 12y + 32 = 0$

$(y - 4)(y - 8) = 0$

Solve for y: $y = 4 \text{ or } 8$

Solve equation in terms of 3^x

$3^x = 4 \text{ or } 8$ **[1 Mark]**

Exemplar Exam Question Answer

$3^x = 4 \text{ or } 8$

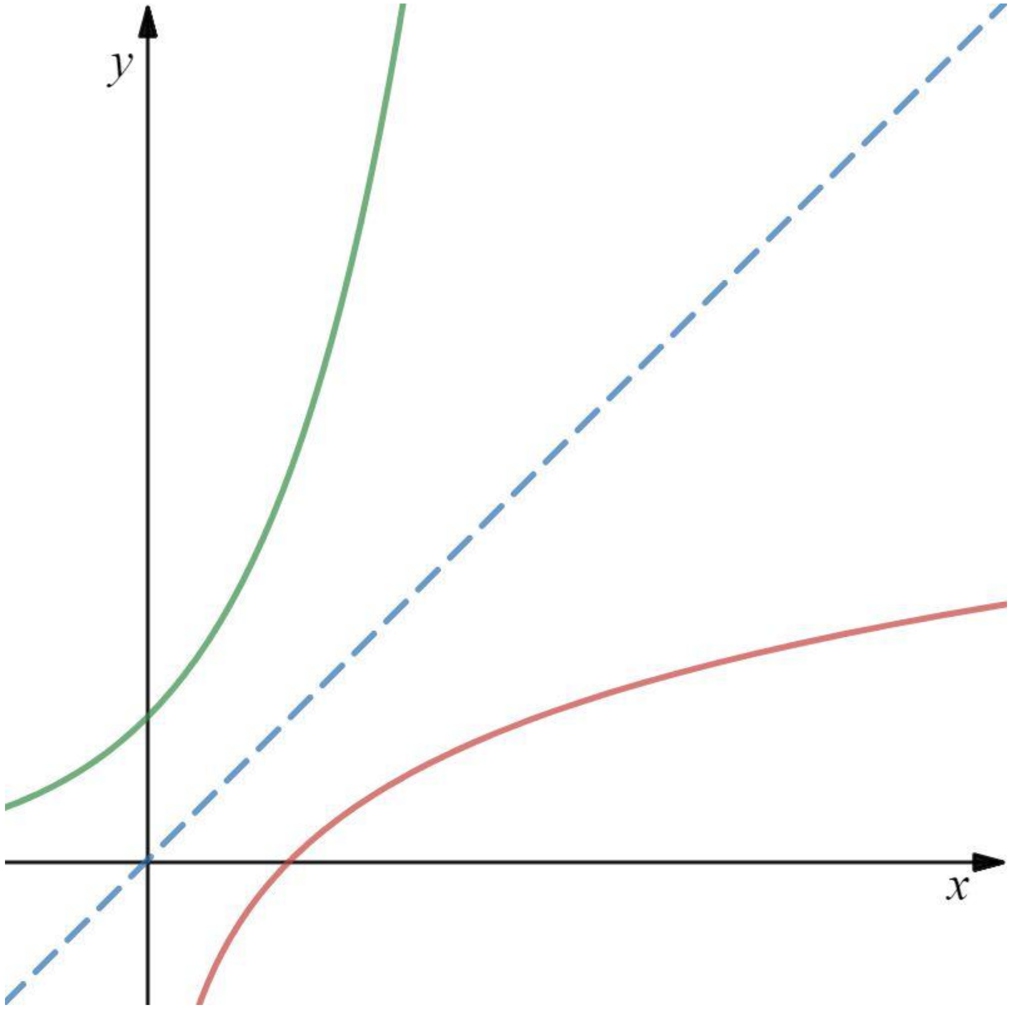
Use logs to solve for x

Take logarithms of both sides:

$x = \log_3 4 \text{ or } \log_3 8$

$x = 1.26 \text{ or } 1.89$ **[1 Mark]**

Exponential and Logarithmic Graphs



Specification Points - AQA

	Content
F1	Know and use the function a^x and its graph, where a is positive. Know and use the function e^x and its graph.

	Content
F2	Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.

	Content
F3	Know and use the definition of $\log_a x$ as the inverse of a^x , where a is positive and $x \geq 0$ Know and use the function $\ln x$ and its graph. Know and use $\ln x$ as the inverse function of e^x

Specification Points – OCR A

1.06 Exponentials and Logarithms		
1.06a	Properties of the exponential function	a) Know and use the function a^x and its graph, where a is positive. Know and use the function e^x and its graph. *Examples may include the comparison of two population models or models in a biological or financial context. The link with geometric sequences may also be made.*
1.06b	Gradient of e^{kx}	b) Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications. *See 1.07j for explicit differentiation of* e^x.
1.06d		d) Know and use the function $\ln x$ and its graph.

Specification Points – OCR MEI

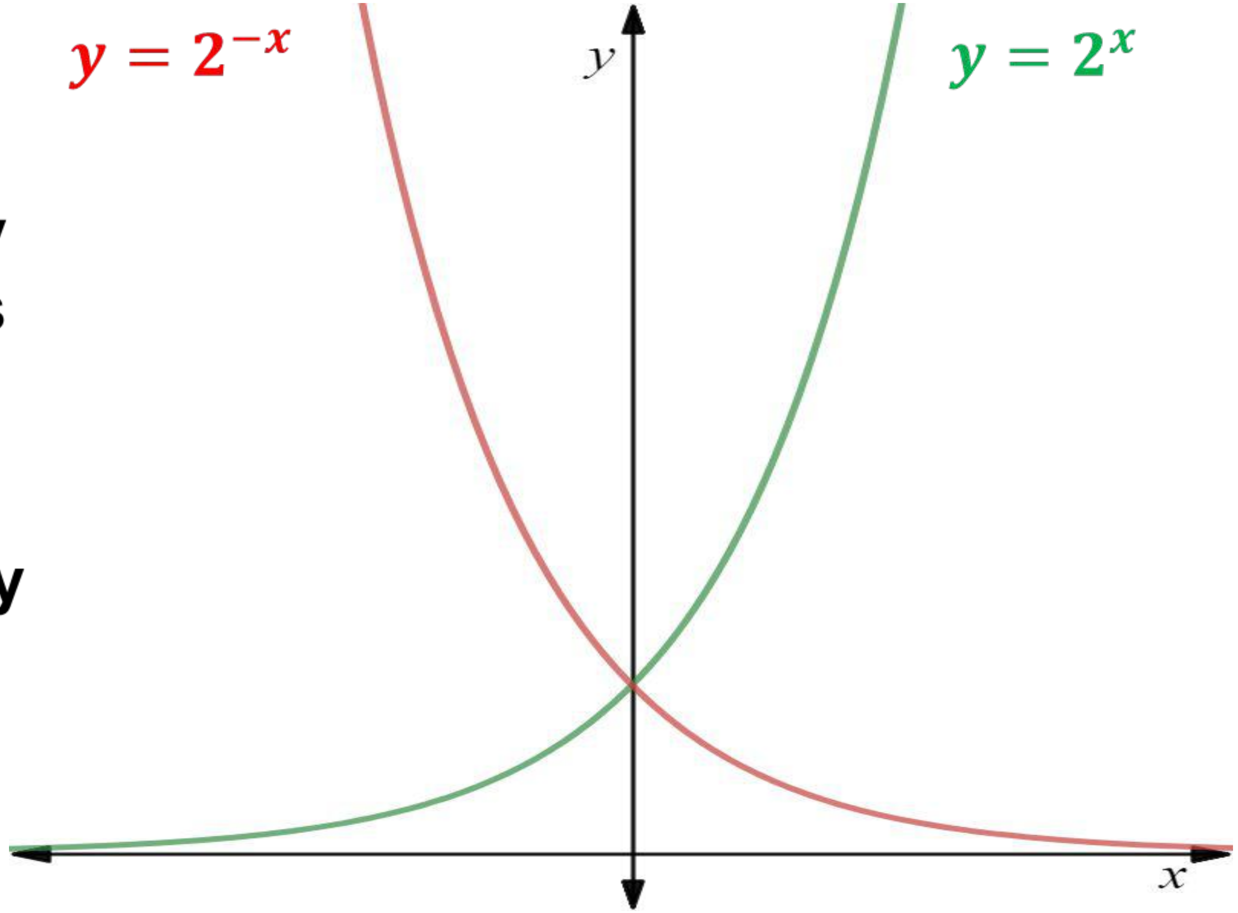
ME1	Know and use the function $y = a^x$ and its graph.	For $a > 0$.
ME8	Know and be able to use the function $y = e^x$ and its graph.	
E9	Know that the gradient of e^{kx} is ke^{kx} and hence understand why the exponential model is suitable in many applications.	
E10	Know and be able to use the function $y = \ln x$ and its graph. Know the relationship between $\ln x$ and e^x.	$\ln x$ is the inverse function of e^x.

Specification Points - Edexcel

6.1	**Know and use the function a^x and its graph, where a is positive.** **Know and use the function e^x and its graph.**	**Understand the difference in shape between $a < 1$ and $a > 1$** To include the graph of $y = e^{ax+b} + c$
6.2	**Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.**	**Realise that when the rate of change is proportional to the y value, an exponential model should be used.**
6.3	**Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geqslant 0$.** **Know and use the function $\ln x$ and its graph.** **Know and use $\ln x$ as the inverse function of e^x**	$a \neq 1$ **Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.**

Exponential Graphs

All **exponential functions** of the **form** $y = a^x$ have a **gradient** which is **proportional** to their y value.



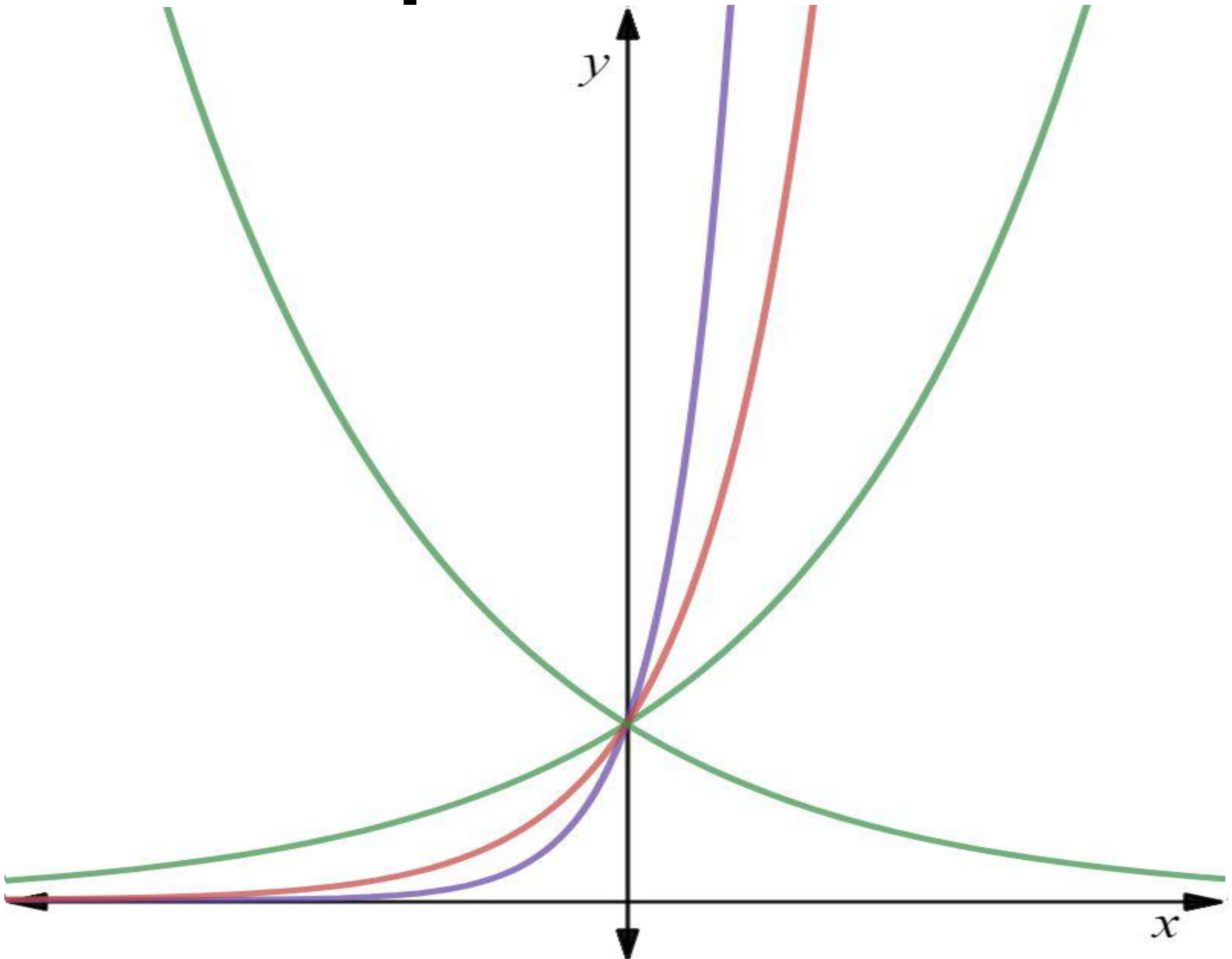
- **Functions** which **increase exponentially** grow at a **rate** which keeps **increasing** as x **increases**.

- **Functions** which **decrease exponentially** decay at a **rate** which keeps **decreasing** as x **increases**.

Exponential Graphs

$y = a^x$

- The **curves** are **steeper** for larger **values** of a.
- The y-intercept $= 1$ for all a.
- The **curve** $y = \left(\frac{1}{a}\right)^x$ looks like $y = a^x$ **reflected** in the y-axis.



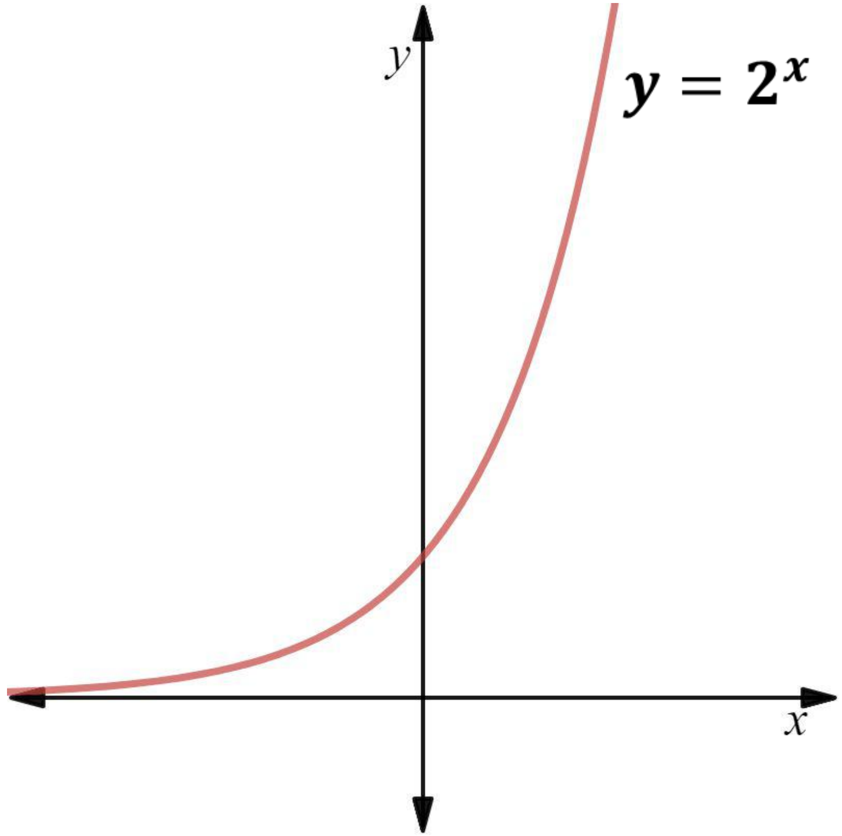
Exemplar Exam Question

Curves are **related** to the one given

1) The following are plots for $y = 2^x$. On the same axes, add sketches for the curves:

(i) $y = 3^x$

(ii) $y = \left(\frac{1}{2}\right)^x$



Nothing exact, just need **key properties**

2 quick and simple parts

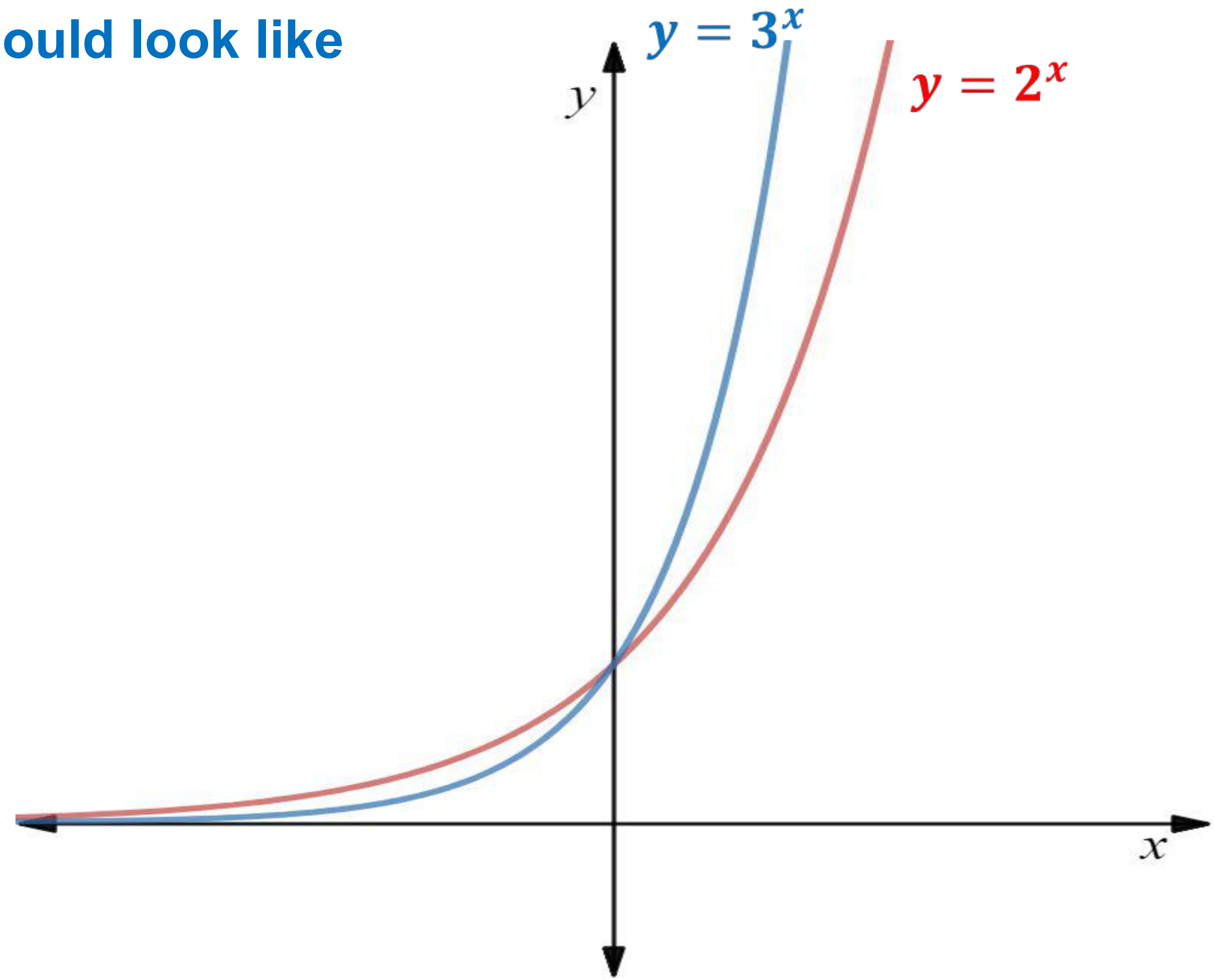
[2 marks]

Exemplar Exam Question Answer

Compare $y = 3^x$ to $y = 2^x$

- Functions are of same form, so similar shapes
- $3 > 2$ so new curve should be steeper
- Both curves will intersect y-axis at same point as give same result for $x = 0$.

Graph should look like



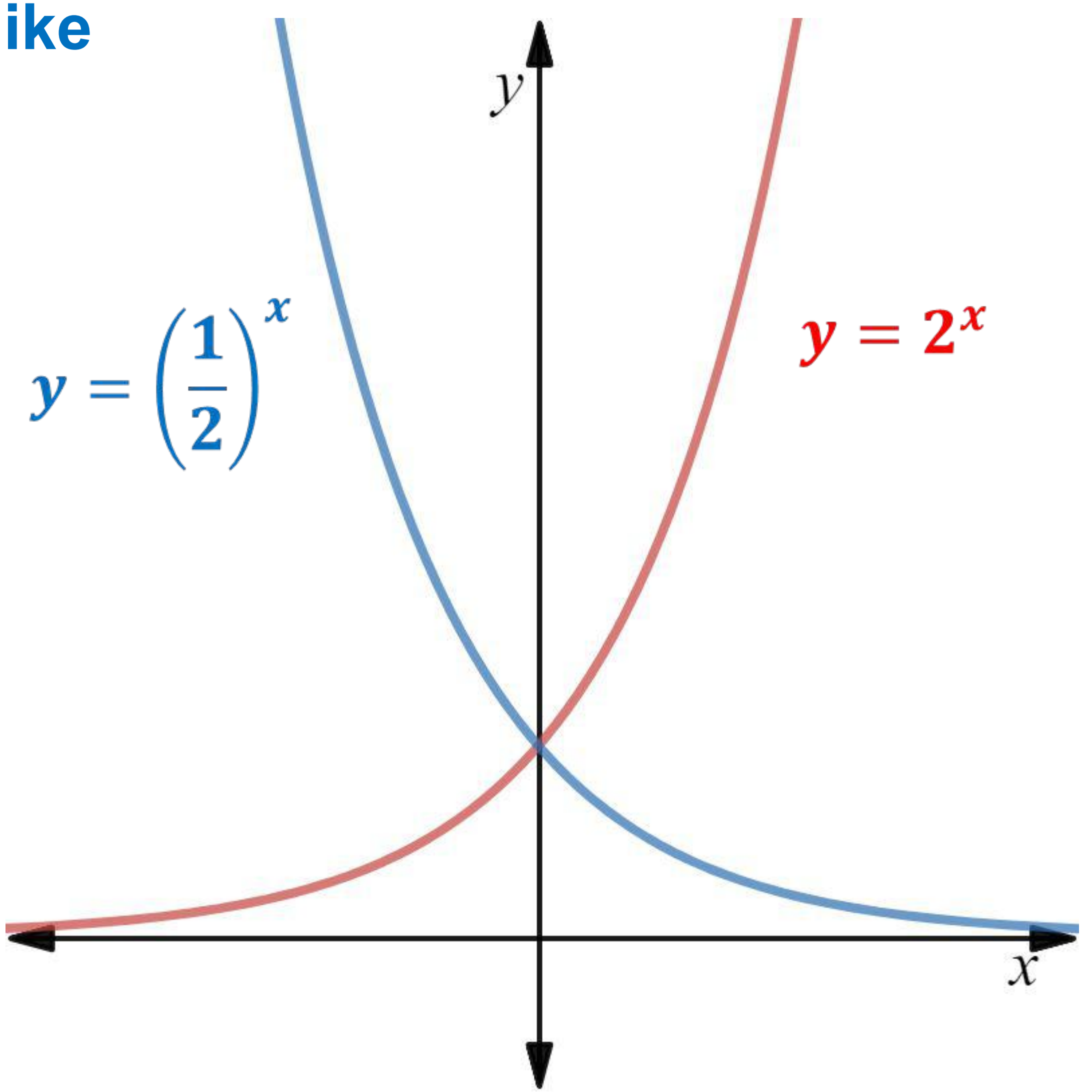
[1 Mark]

Exemplar Exam Question Answer

Remember rules for inverse graph

$y = \left(\frac{1}{2}\right)^x$ will look like $y = 2^x$ reflected in x-axis

Graph should look like

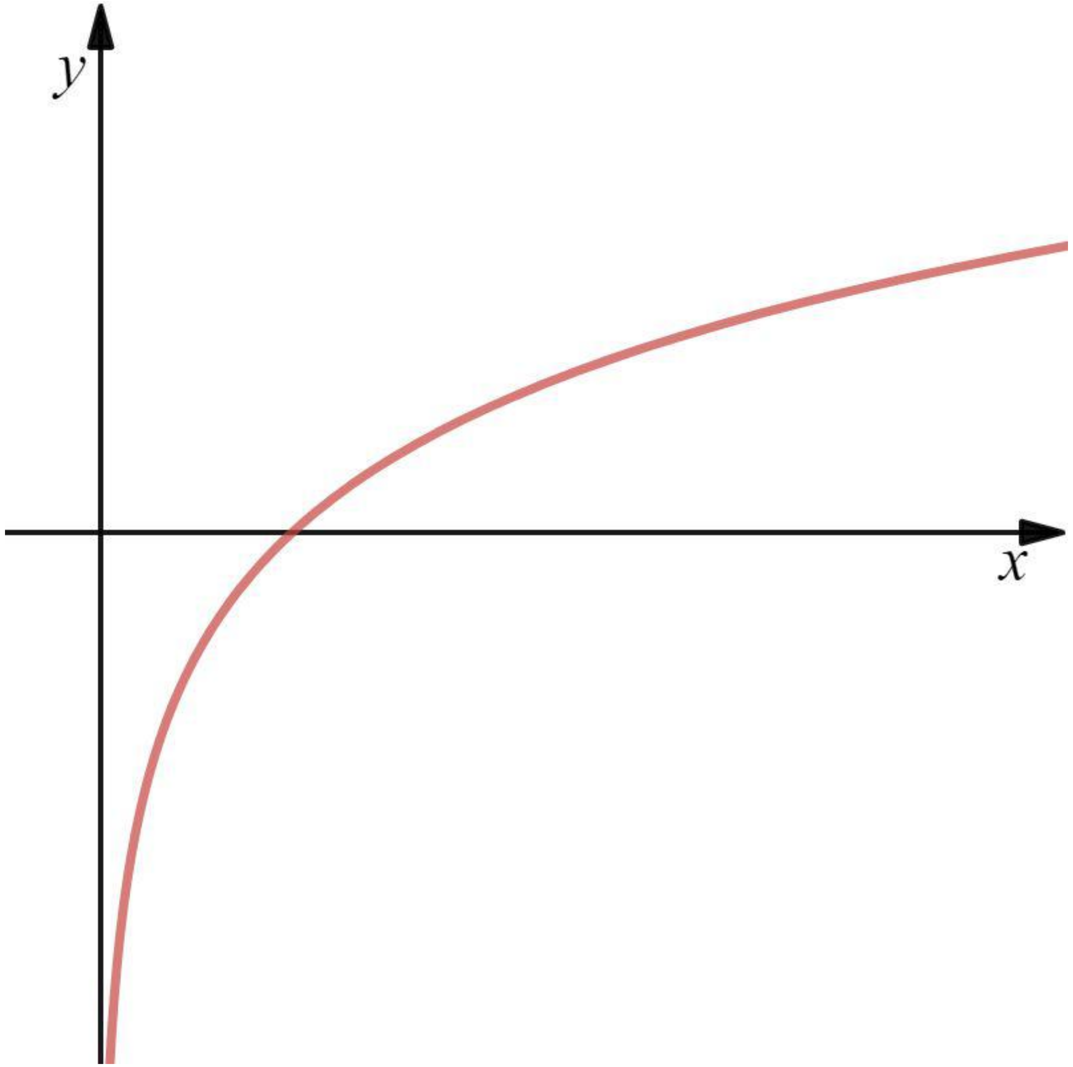


[1 Mark]

Logarithmic Graphs

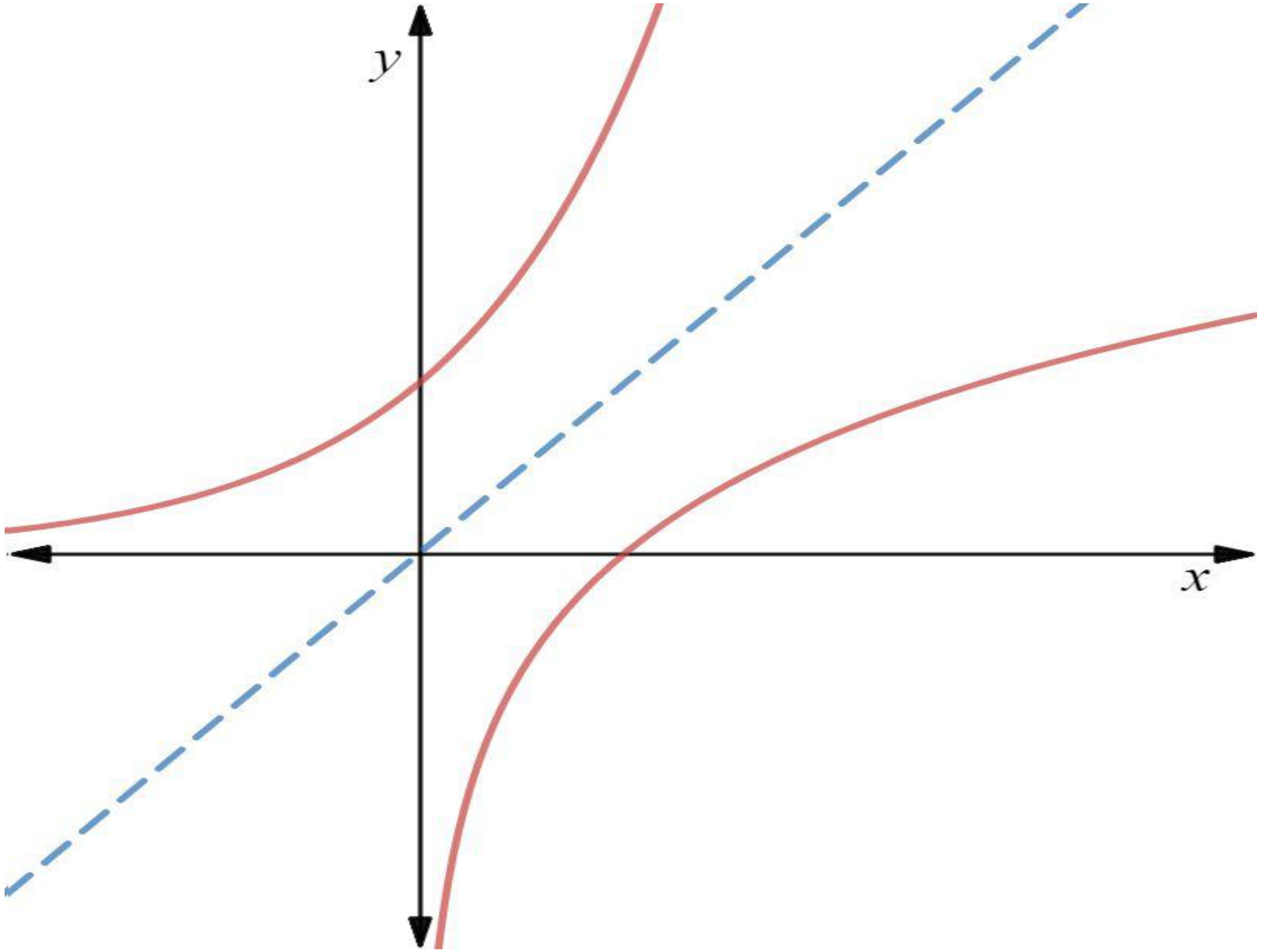
$y = \ln x$

- **Logarithmic functions** cross the x-axis at $x = 1$.
- The **graph asymptotically** approaches the y-axis as x tends to 0.



Logarithmic Graphs

The **graph** of $y = \ln x$ is the **reflection** of the **graph** $y = e^x$ in the **line** $y = x$.



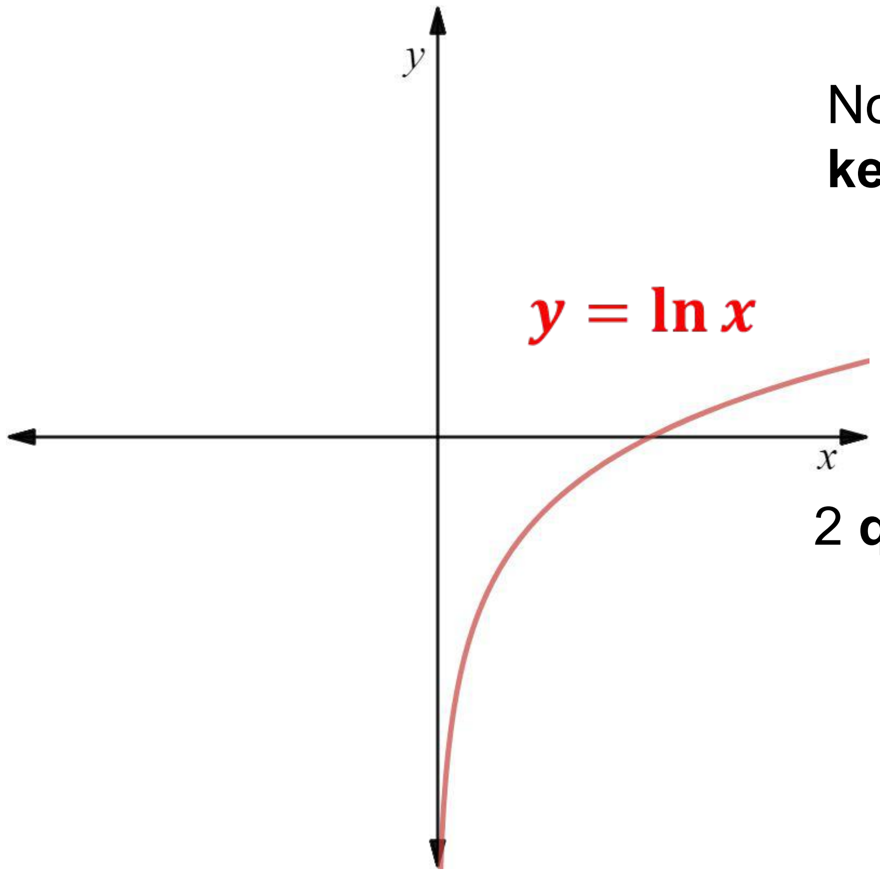
Exemplar Exam Question

Curves are **related** to one given

1) The following are plots of $y = \ln x$. On the same axes sketch curves for

(i) $y = e^x$

(ii) $y = \ln(x + 2)$



Nothing exact, just need **key properties**

2 **quick and simple** parts

You should show any **points** where your **curve intersects** the axes

[2 marks]

Exemplar Exam Question Answer

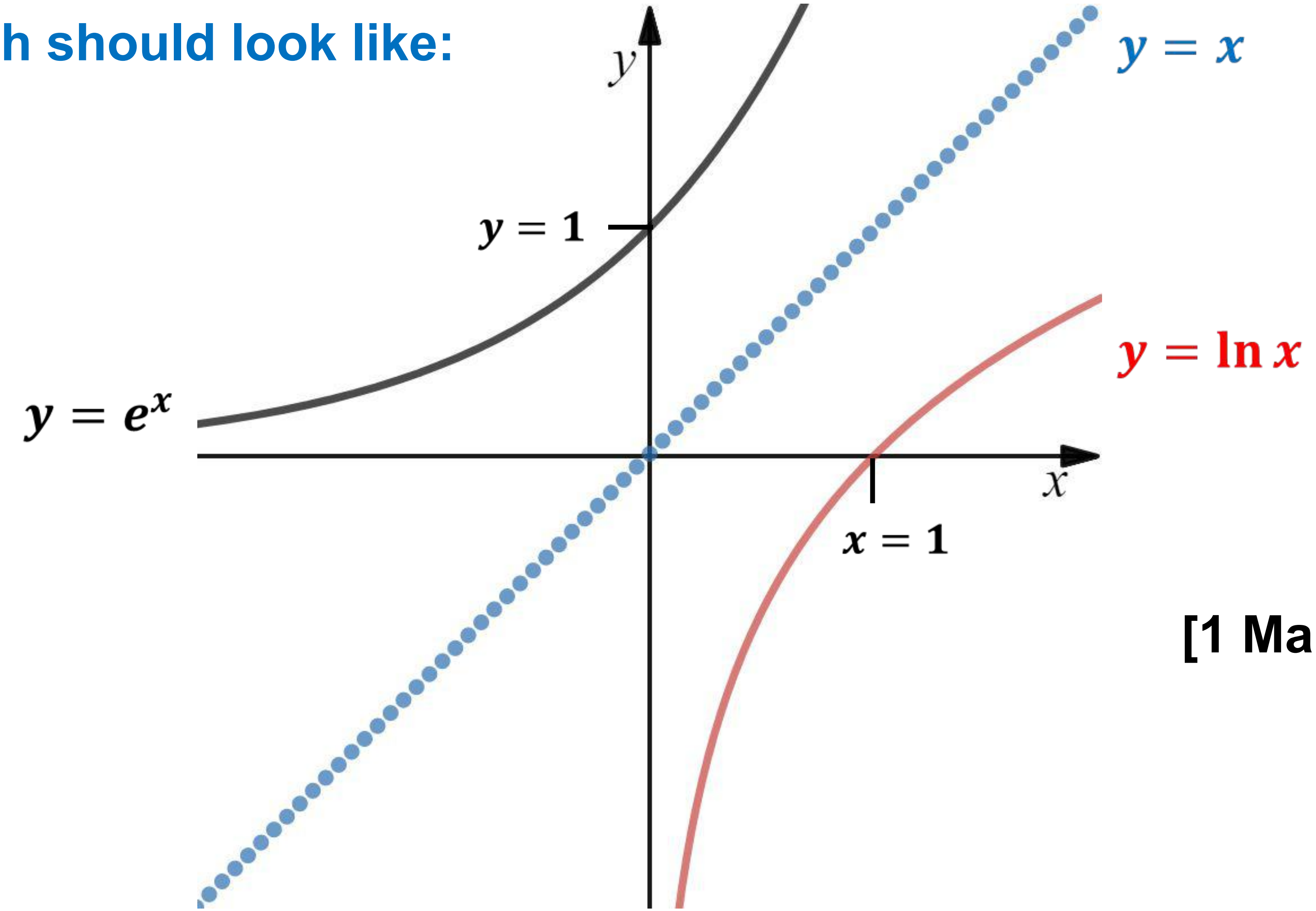
(i) $y = e^x$

Use relationship between e^x and $\ln x$

e^x is inverse of $\ln x$

So graph is reflected in line $y = x$

New graph should look like:



[1 Mark]

Exemplar Exam Question Answer

(ii) $y = \ln(x+2)$

Use transformation properties

$f(x+2)$ is $f(x)$ translated left by 2

Therefore $\ln(x+2)$ is $\ln(x)$ translated left by 2

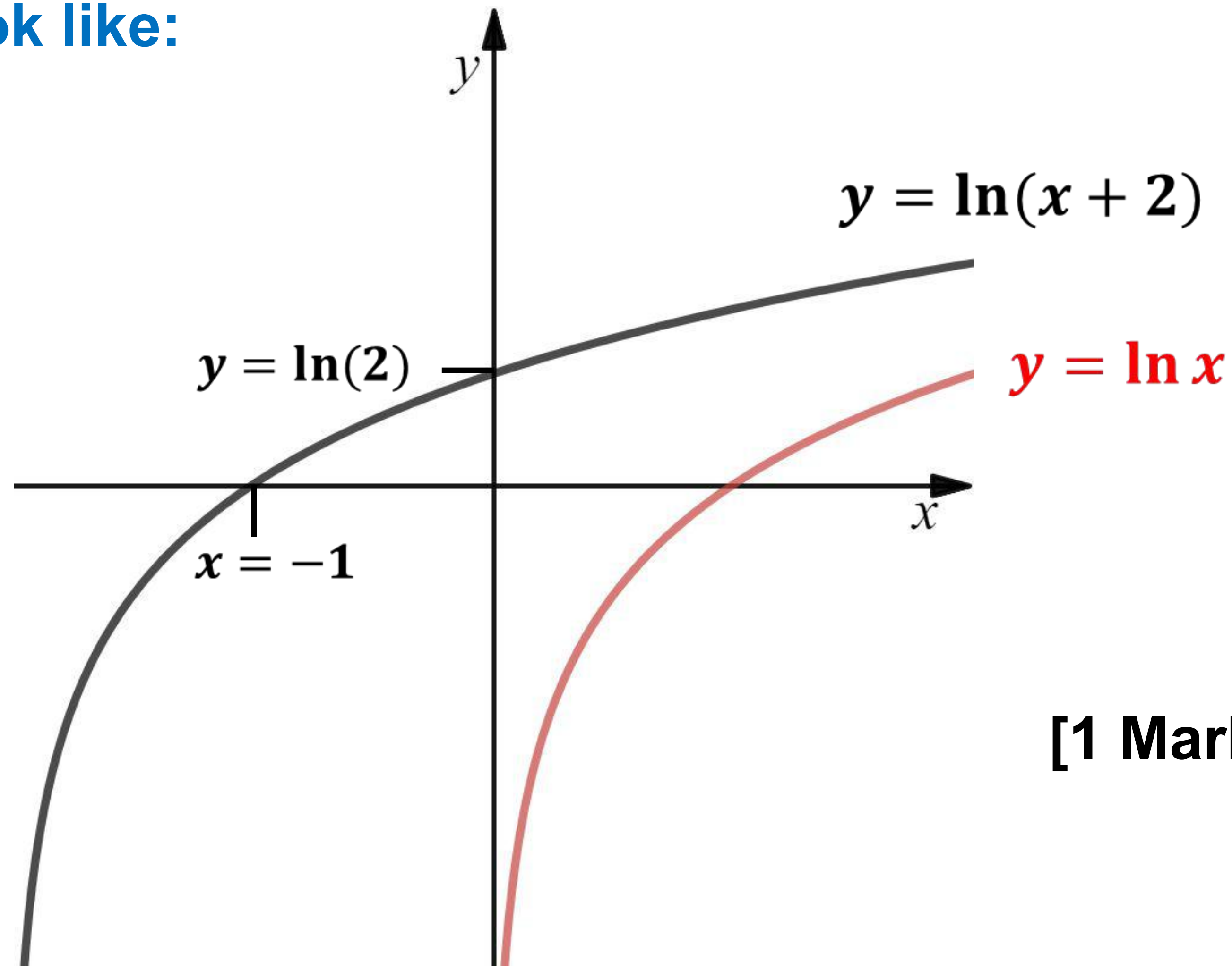
Determine intersects

$\ln x$ intersects x-axis at $x = 1$

So $\ln(x+2)$ intersects x-axis at $x = 1 - 2 = -1$

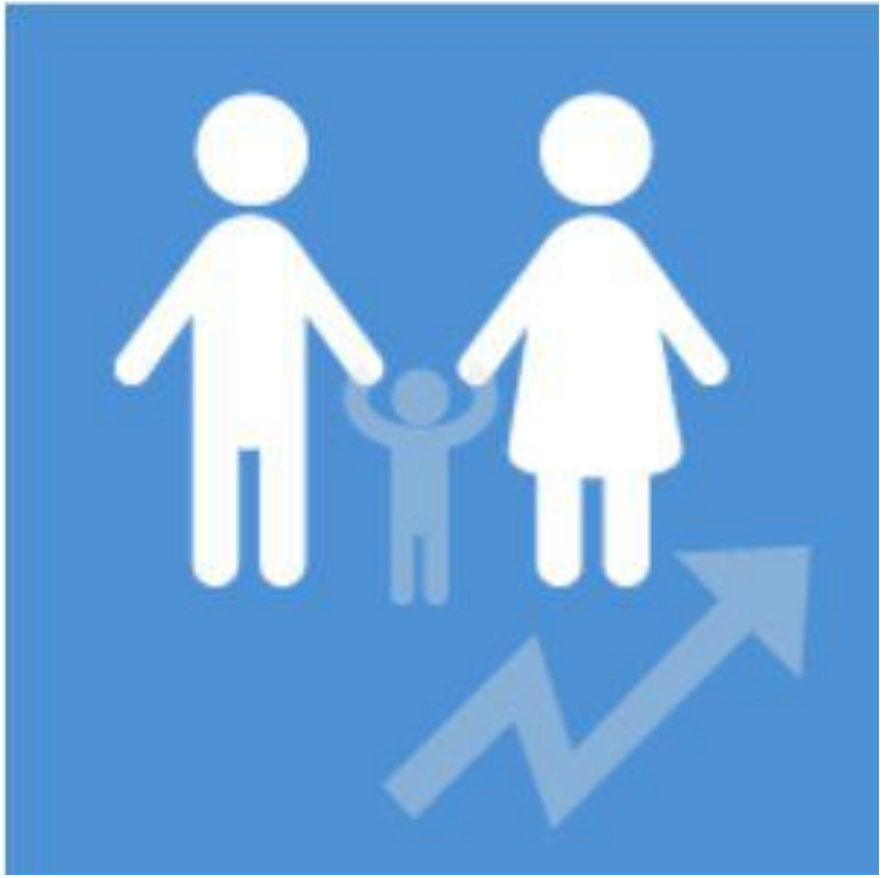
y-intersect is when $x = 0 \Rightarrow y$-intersect is $\ln 2$

New graph should look like:



[1 Mark]

Exponential Processes



Specification Points - AQA

	Content
F6	Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.

	Content
F7	Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.

Specification Points – OCR A

1.06i	Modelling using exponential functions	i) Understand and be able to use exponential growth and decay and use the exponential function in modelling. *Examples may include the use of* e *in continuous compound interest, radioactive decay, drug concentration decay and exponential growth as a model for population growth. Includes consideration of limitations and refinements of exponential models.*
1.06h	Reduction to linear form	h) Be able to use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y. *Learners should be able to reduce equations of these forms to a linear form and hence estimate values of* a *and* n*, or* k *and* b *by drawing graphs using given experimental data and using appropriate calculator functions.*

Specification Points – OCR MEI

Exponential growth and decay	E11	Be able to solve problems involving exponential growth and decay; be able to consider limitations and refinements of exponential growth and decay models.	Understand and use exponential growth and decay: use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models. Finding long term values.
	E7	Know how to reduce the equations $y = ax^n$ and $y = ab^x$ to linear form and, using experimental data, to use a graph to estimate values of the parameters.	By taking logarithms of both sides and comparing with the equation $y = mx + c$. Learners may be given graphs and asked to select an appropriate model.

Specification Points - Edexcel

6 **Exponentials and logarithms** *continued*	6.7	**Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.**	**Students may be asked to find the constants used in a model.** **They need to be familiar with terms such as initial, meaning when $t = 0$.** **They may need to explore the behaviour for large values of t or to consider whether the range of values predicted is appropriate.** **Consideration of a second improved model may be required.**
	6.6	**Use logarithmic graphs to estimate parameters in relationships of the form** $y = ax^n$ **and** $y = kb^x$**, given data for x and y**	**Plot $\log y$ against $\log x$ and obtain a straight line where the intercept is $\log a$ and the gradient is n** **Plot $\log y$ against x and obtain a straight line where the intercept is $\log k$ and the gradient is $\log b$**

Reduction to Linear Form

If x and y have a relationship given by $y = ab^x$ we can use a **table** of **values** for x and y and **logarithms** to **determine** the **parameters** a and b.

$$y = ab^x$$

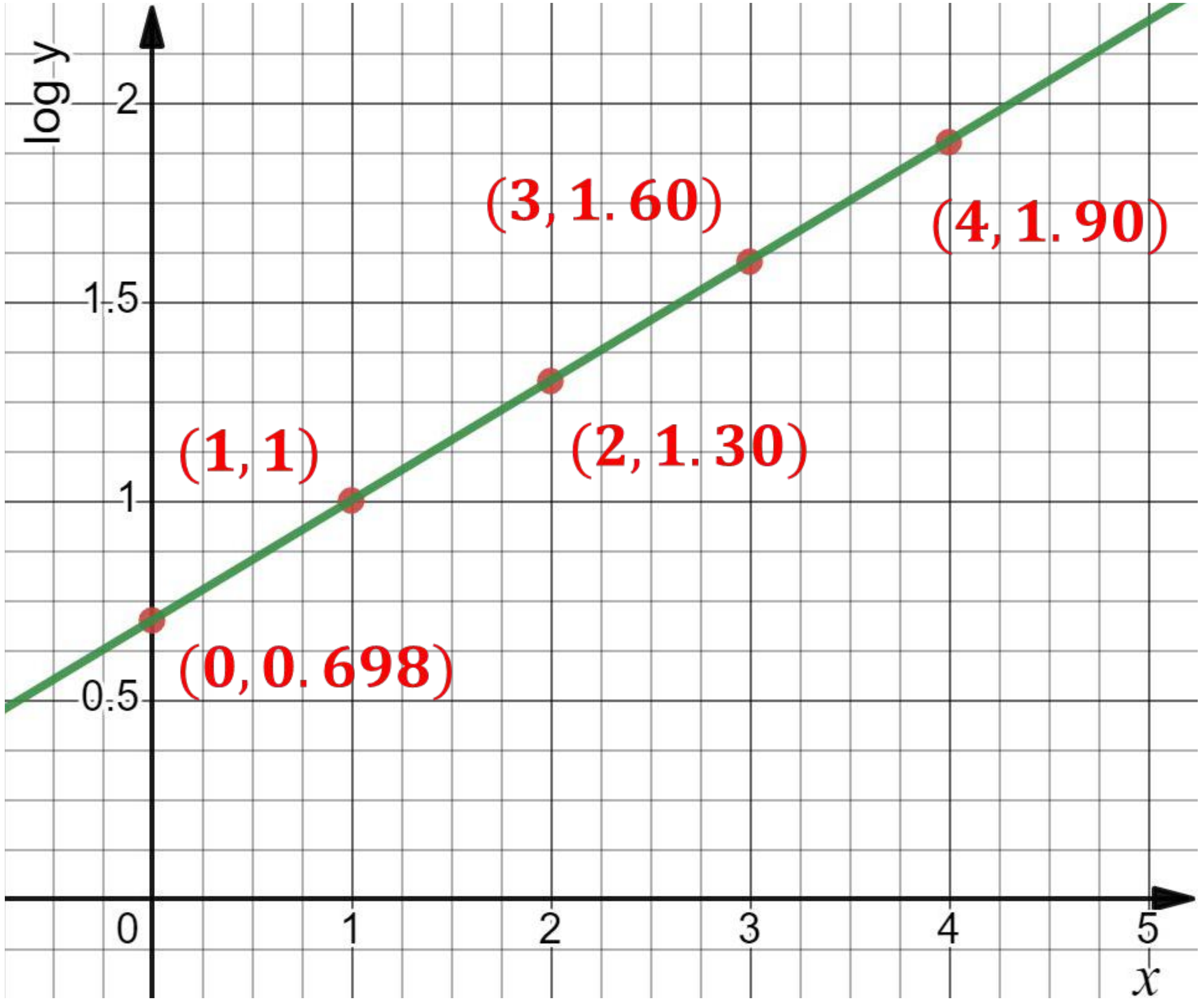
- Take **logarithms** of **both sides**.
- **Compare** to the **equation** for a **straight line**.
- **Therefore** we can **plot** a **linear graph** of $\log y$ against x to **estimate** the **value** of b and a from the **gradient** and **y-intercept respectively**.

$$y = mx + c$$

Reduction to Linear Form

$$y = ab^x$$

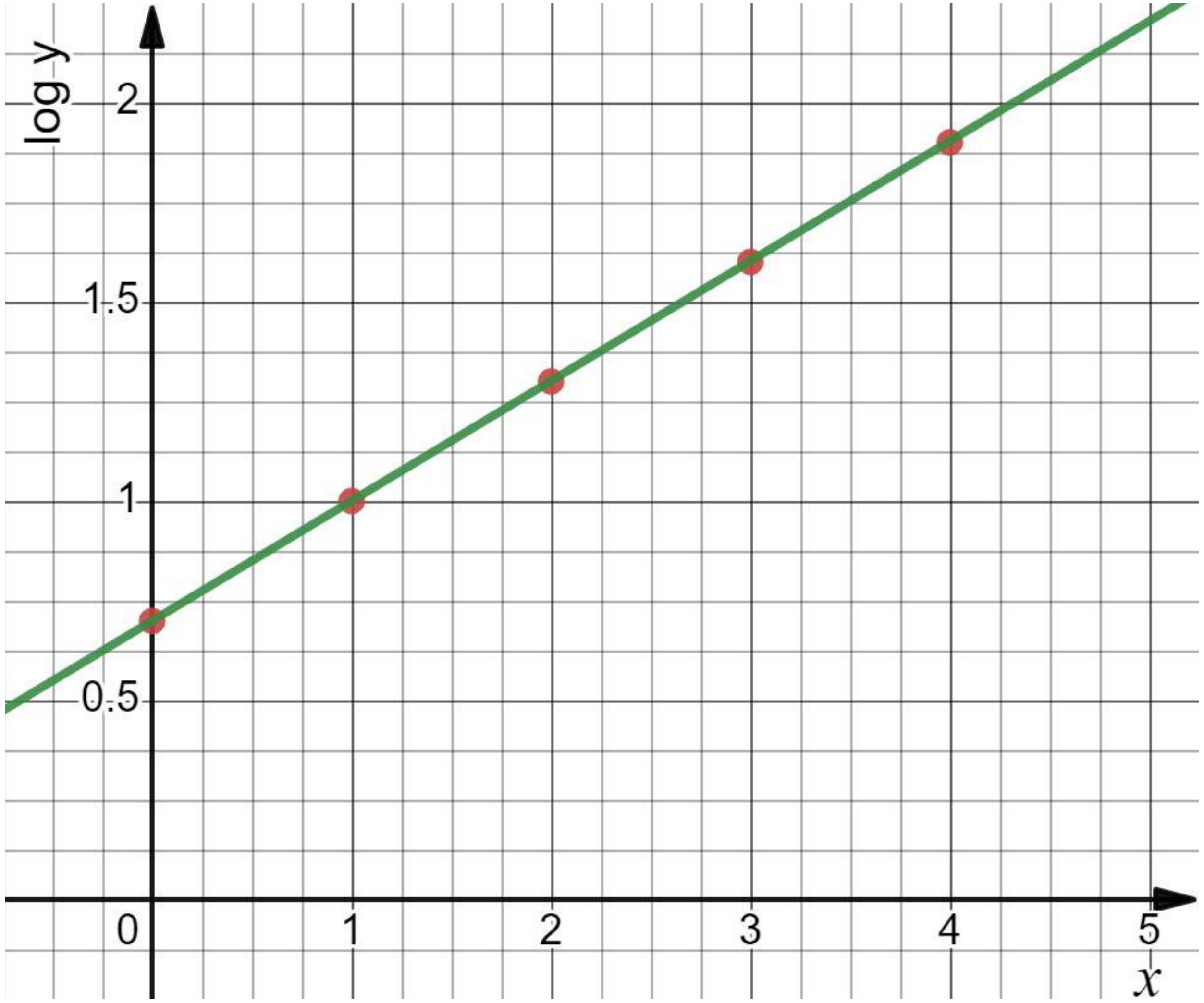
x	y	$\log_{10} y$
0	5	0.698 ...
1	10	1
2	20	1.30 ...
3	40	1.60 ...
4	80	1.90 ...



Reduction to Linear Form

$$y = ab^x$$

$$y = mx + c$$



Reduction to Linear Form

If x and y have a relationship given by $y = px^n$ we can use a **table** of **values** for x and y and **logarithms** to **determine** the **parameters** p and n.

$$y = px^n$$

- Take **logarithms** of **both sides**.
- **Compare** to the **equation** for a **straight line**.
- **Therefore** we can **plot** a **linear graph** of $\log y$ against $\log x$ to **estimate** the **value** of n and p from the **gradient** and **y-intercept respectively**.

$$y = mx + c$$

Exemplar Exam Question

1) The following graph shows the relationship between $\log_{10} y$ and $\log_{10} x$. Using the graph, determine a relationship between y and x of the form $y = f(x)$ where f is a function of x. You should give any determined constants to the nearest whole number.

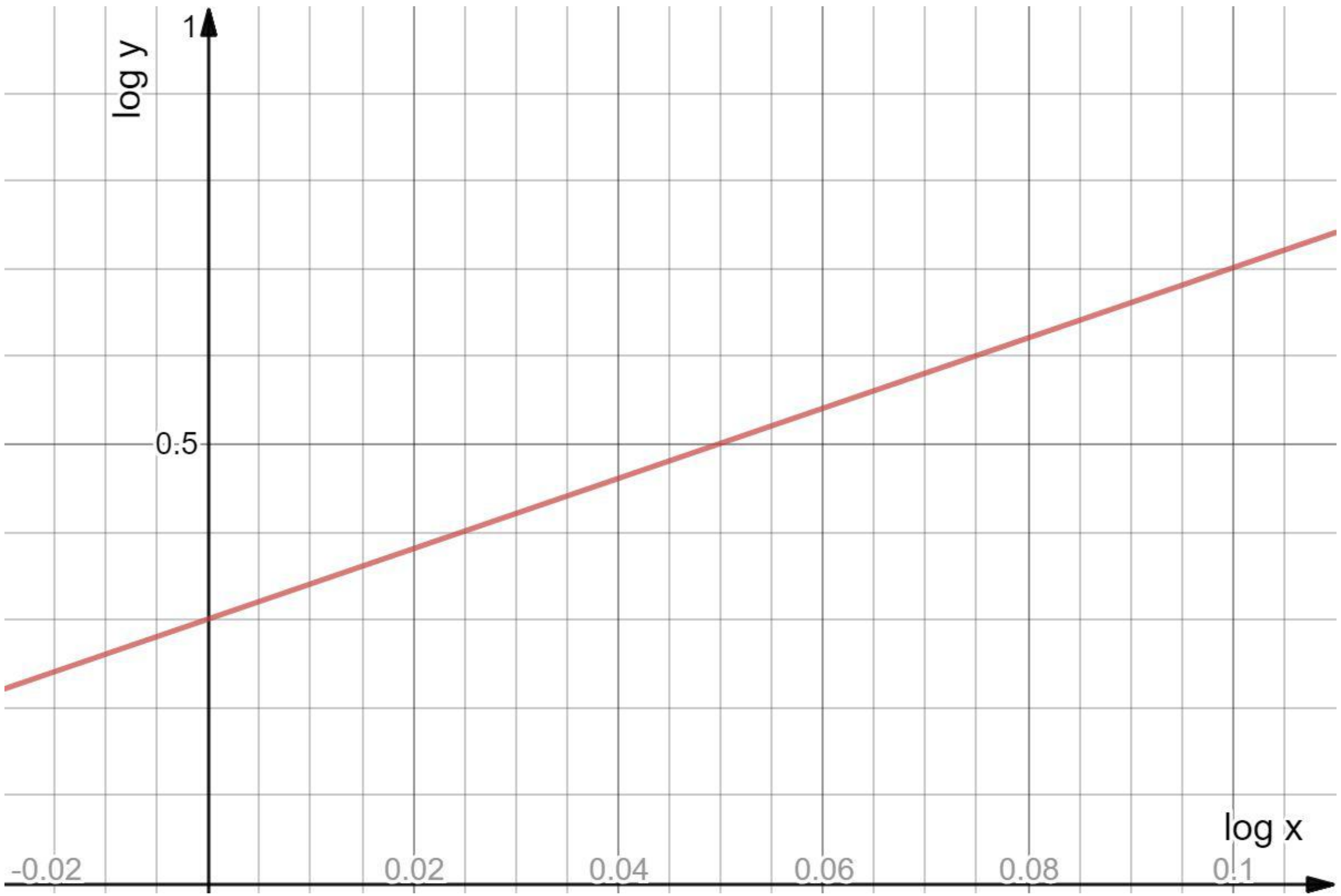
Need to **remove logs**

Can **expect** the **relationship** to be of the **form** $y = px^n$.

[4 marks]

4 stages to solution

Exemplar Exam Question



Exemplar Exam Question Answer

Determine shape of graph

Straight line graph between $\log_{10} y$ and $\log_{10} x$

So graph has equation $\log_{10} y = m \log_{10} x + \log_{10} c$. **[1 Mark]**

Exemplar Exam Question Answer

Find gradient of graph

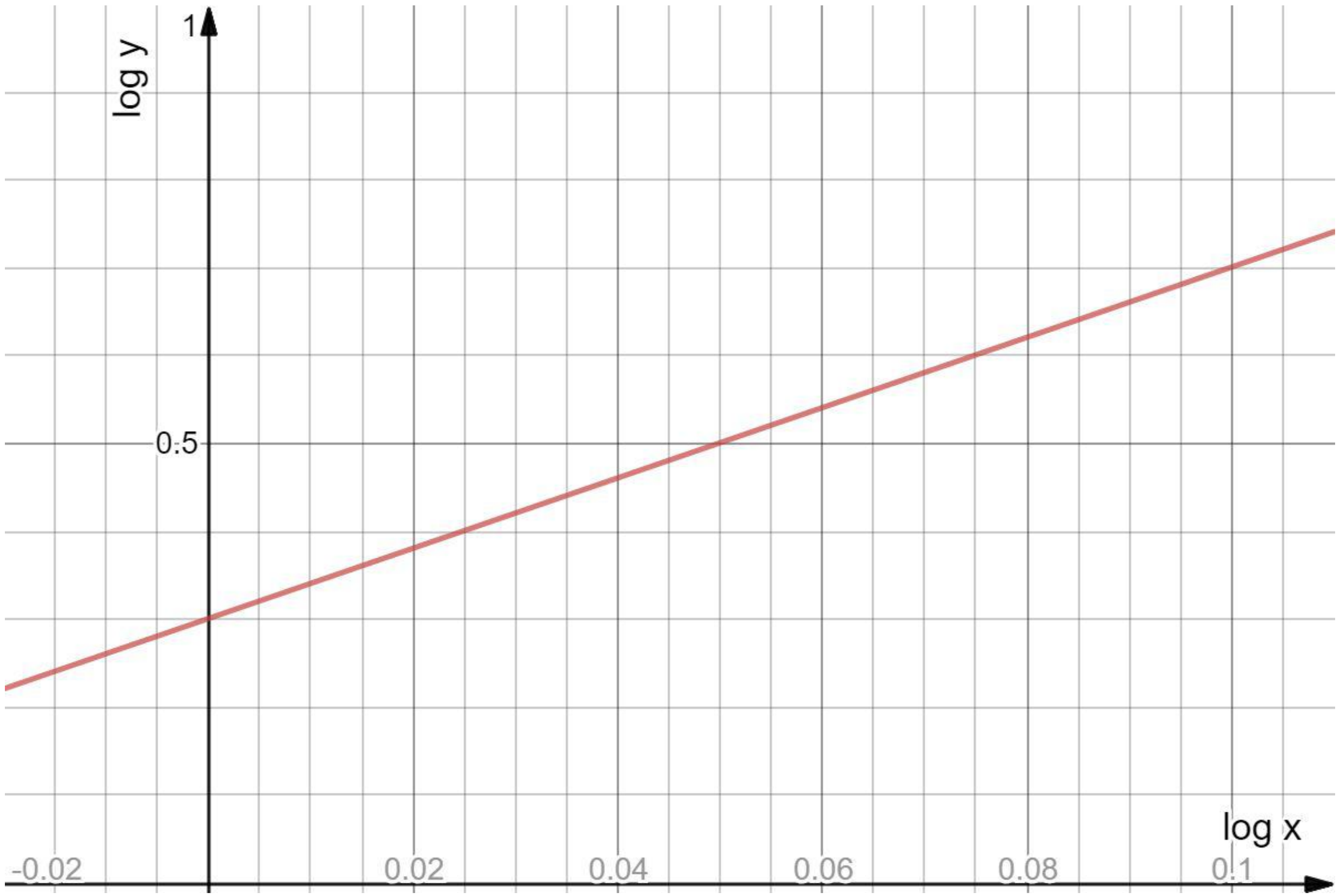
$$m = \frac{\Delta y}{\Delta x} = \frac{0.4}{0.1} = 4$$

[1 Mark]

Find intercept of graph

$$\log_{10} c = 0.3$$

[1 Mark]



Exemplar Exam Question Answer

$\log_{10} y = 4\log_{10} x + 0.3$

Rearrange to form $y = f(x)$

Use log rules to simplify equation

Using $\log_a x^n = n\log_a x$ and $n = \log_a a^n$

$$\log_{10} y = \log_{10} x^4 + \log_{10} 10^{0.3}$$

Using $\log_a x + \log_a y = \log_a(xy)$

$$\log_{10} y = \log_{10}(10^{0.3} \times x^4)$$

Exemplar Exam Question Answer

$\log_{10} y = \log_{10}(10^{0.3} \times x^4)$

Remove logs to find needed equation

$y = 10^{0.3} \times x^4$

Write to nearest whole numbers

$y = 2x^4$

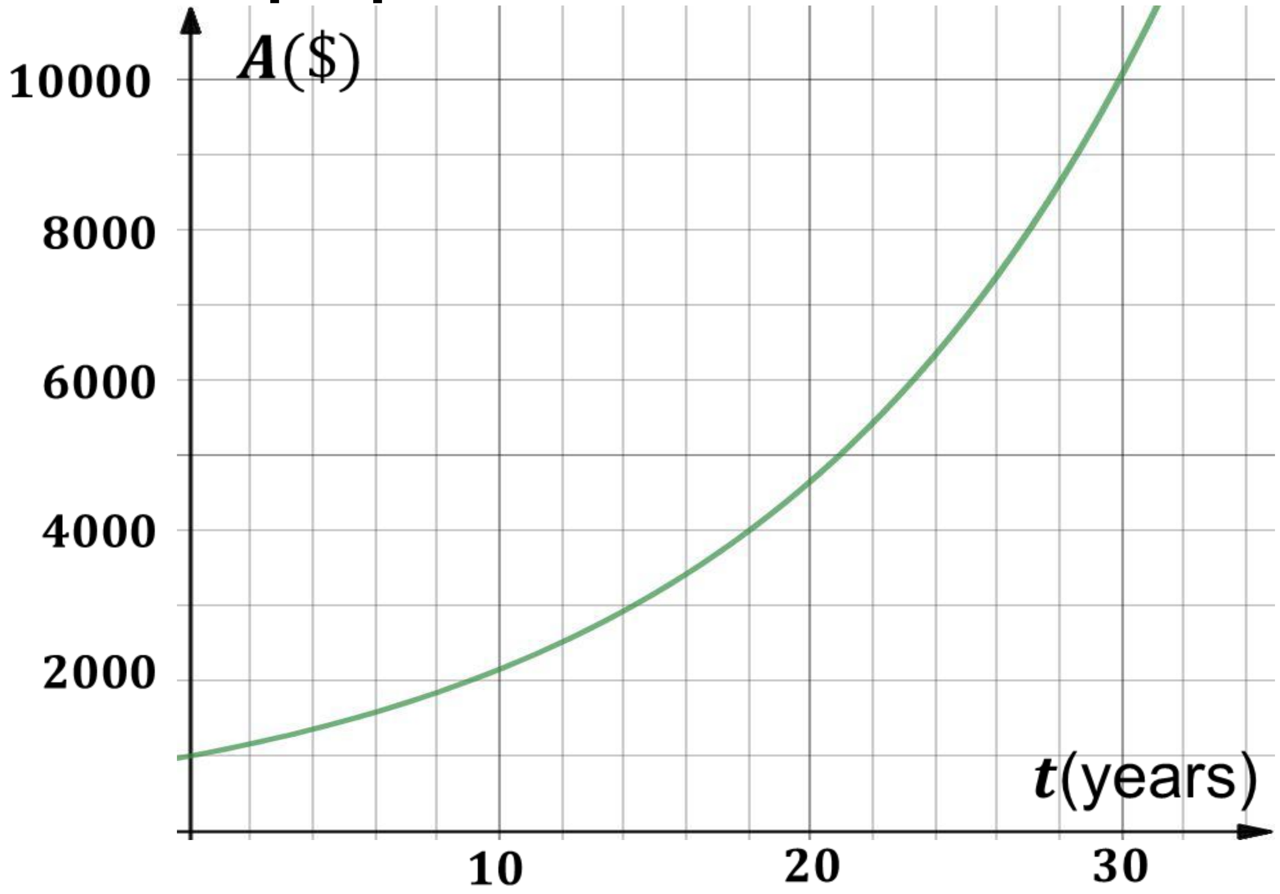
[1 Mark]

Exponential Growth and Decay

Many **real-life processes** can be modelled by **exponential functions** as they also have a **rate of increase** or **decrease** which is **proportional** to their **size**.

Continuous compound interest:

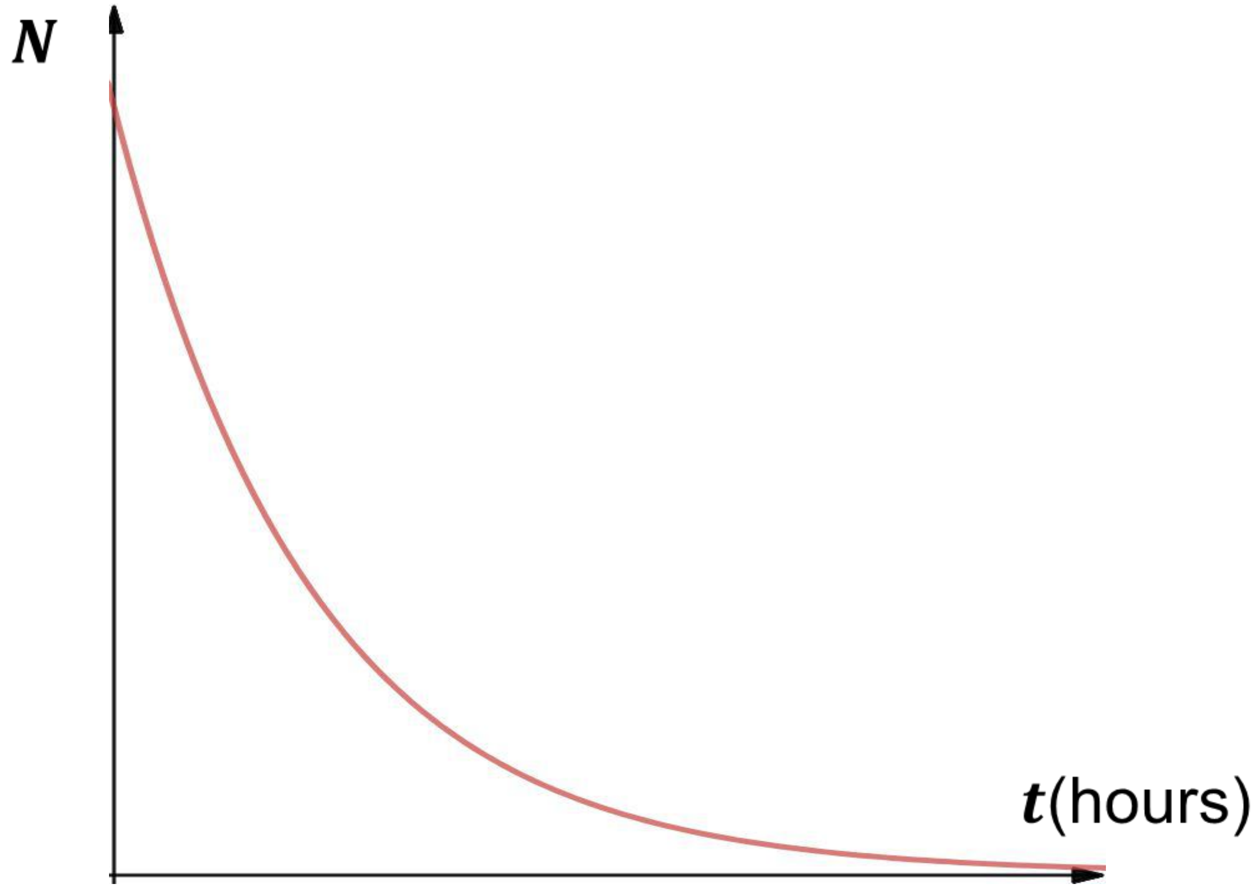
$$A = P\left(1 + \frac{r}{100}\right)^t$$



Exponential Growth and Decay

Radioactive decay:

$$N = 8000e^{-\lambda t}$$



- Does not **apply** at **small values** of N as each **individual decay** occurs due to **random chance**.

Exemplar Exam Question

Real world question on **population dynamics** and **exponential models**

1) The Mayor of London has determined a need to track the city's pigeon population. In 2012 there were an estimated one million (1,000,000) pigeons in London while in 2018 there were an estimated one billion (1,000,000,000) pigeons in London.
A model of the form $N = ne^{kt}$ is suggested, with t the number of years since 2012 and k a constant. Use the given data to calculate suitable values for n and k. Give your value for k to 3 significant figures

[3 marks]

Two values required, do we need a simultaneous equation or are one of these values eliminated for a certain t?

Long complicated result, need to **round**

3 **steps** expected

Exemplar Exam Question Answer

$$N = ne^{kt}$$

List what information is given

$$N = \text{Number of Pigeons}$$

$$N_1 = 1{,}000{,}000$$
$$N_2 = 1{,}000{,}000{,}000$$

$$t = \text{Year} - 2012$$

$$t_1 = 2012 - 2012 = 0$$
$$t_2 = 2018 - 2012 = 6$$

Exemplar Exam Question Answer

$N = ne^{kt}$

Use known data to find n

Let $t = t_1 = 0$ and $N = N_1 = 1,000,000$

So

$$1,000,000 = ne^{k \times 0}$$

$$\Rightarrow n = 1,000,000$$

[1 Mark]

Exemplar Exam Question Answer

$N = 1,000,000e^{kt}$

Use known data to find k

Let $t = t_2 = 6$ and $N = N_2 = 1,000,000,000$

So

$$1,000,000,000 = 1,000,000e^{6k}$$

$$1,000 = e^{6k}$$

[1 Mark]

Exemplar Exam Question Answer

Apply logarithms and calculate answer

$$\ln 1,000 = 6k$$

$$k = \frac{1}{6}\ln 1,000 = 1.1512925\ldots$$

$$= 1.15\ \mathrm{s}^{-1}\ (\text{to } 3\ \text{s.f.})$$

[1 Mark]

MINI MOCK PAPER

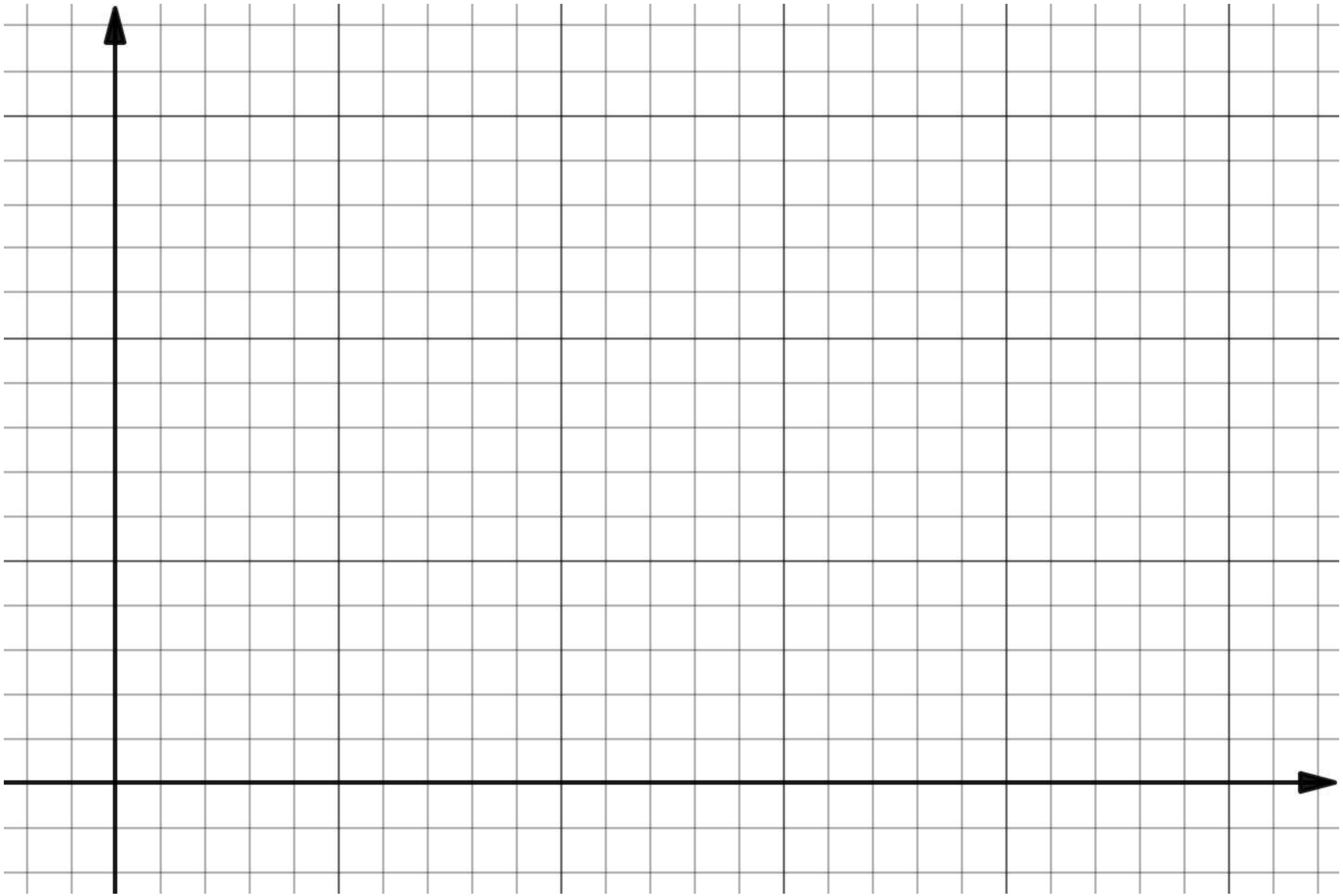
Exam Question

1) A scientific experiment was carried out numerous times by a group of students to determine a relationship between two variables x and y. The average of their results are printed in the table below.

2	32
3	108
4	256
5	500

The students have deduced that y is related to x by the equation $y = Ax^n$. Use the data and the gridlines provided to estimate the values of A and n to the nearest whole number.

[6 marks]



Exam Question Answer

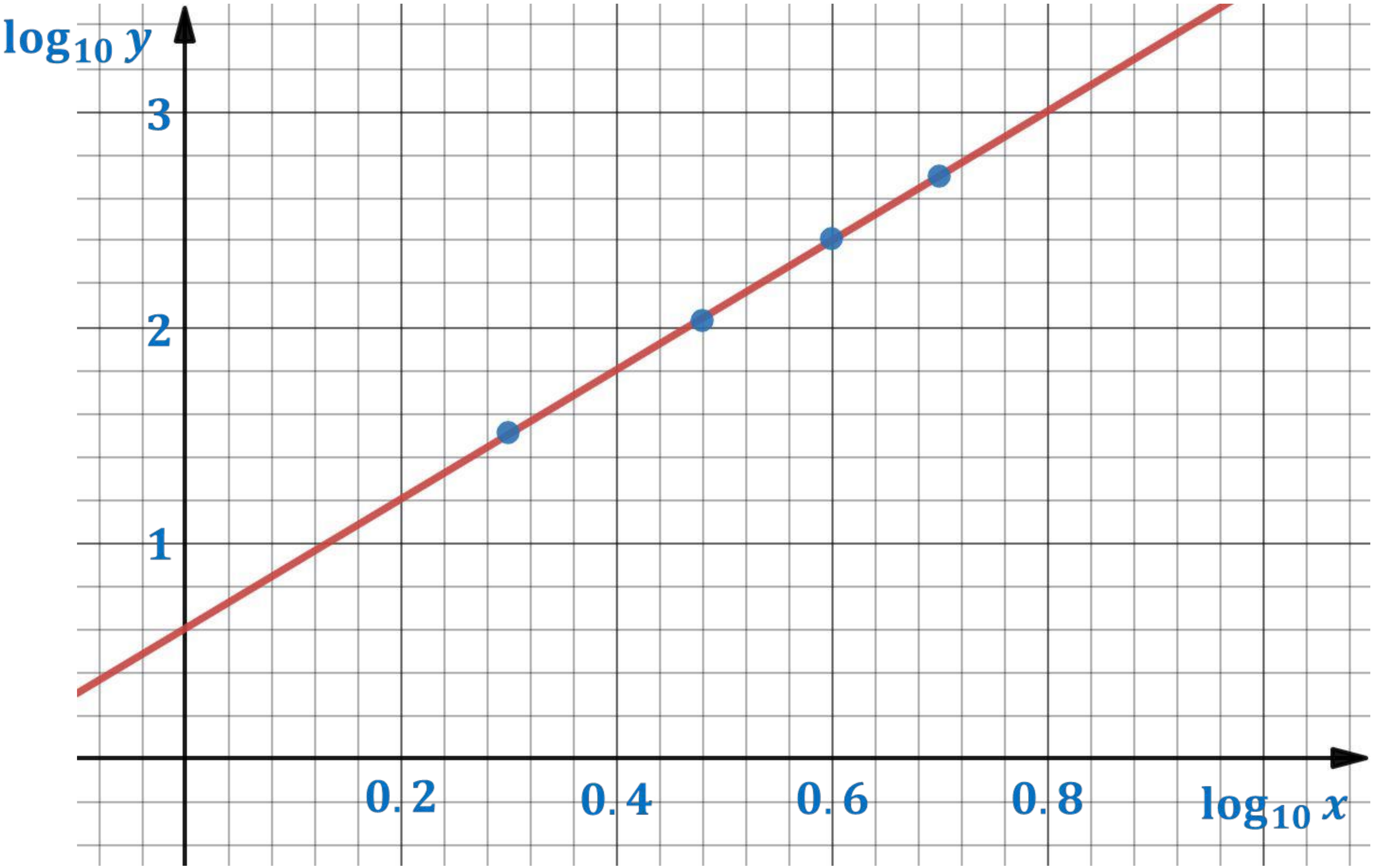
Convert all values to $\log_{10}$ of value

Quickly calculate one by one

0.30	1.51
0.48	2.03
0.60	2.41
0.70	2.70

[1 Mark]

Now use these values to plot graph



[1 Mark]

Exam Question Answer

Find gradient and intercept of linear graph

Intercept where graph intercepts y axis

Intercept $c \approx 0.60$

[1 Mark]

Gradient calculated from curve
Select e.g. (0, 0.6), (0.8, 3.0)

$$\text{Gradient } m \approx \frac{3.0 - 0.6}{0.8} \approx 3$$

[1 Mark]

Exam Question Answer

Find relation between linear graph and required constants

Take log of given equation

$$y = Ax^n$$
$$\log_{10} y = \log_{10} Ax^n$$
$$\log_{10} y = \log_{10} A + n \log_{10} x$$

[1 Mark]

State/calculate constants from graph info

$$n = m = 3$$
$$\log_{10} A \approx c,\ C = 10^A \approx 4$$

[1 Mark]