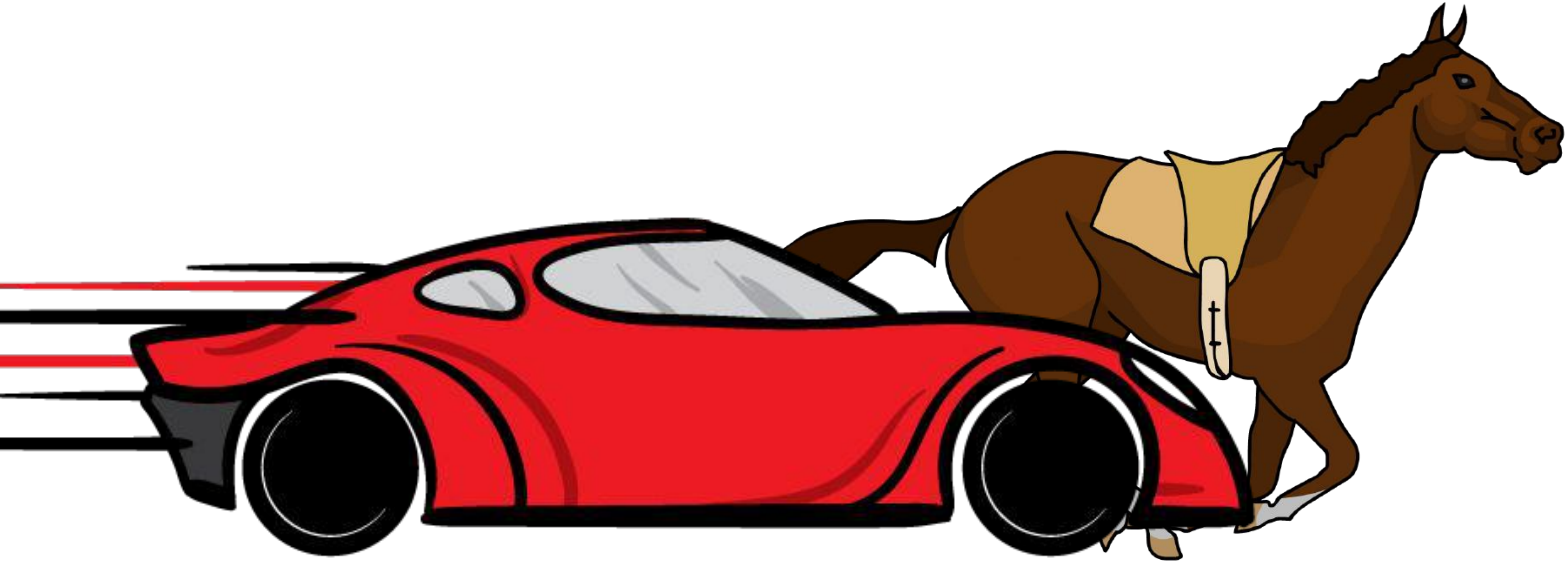
Mechanics:
Vectors, Kinematics & Forces



Material Covered

Vectors

1. Vector Notation
2. Resolving Vectors

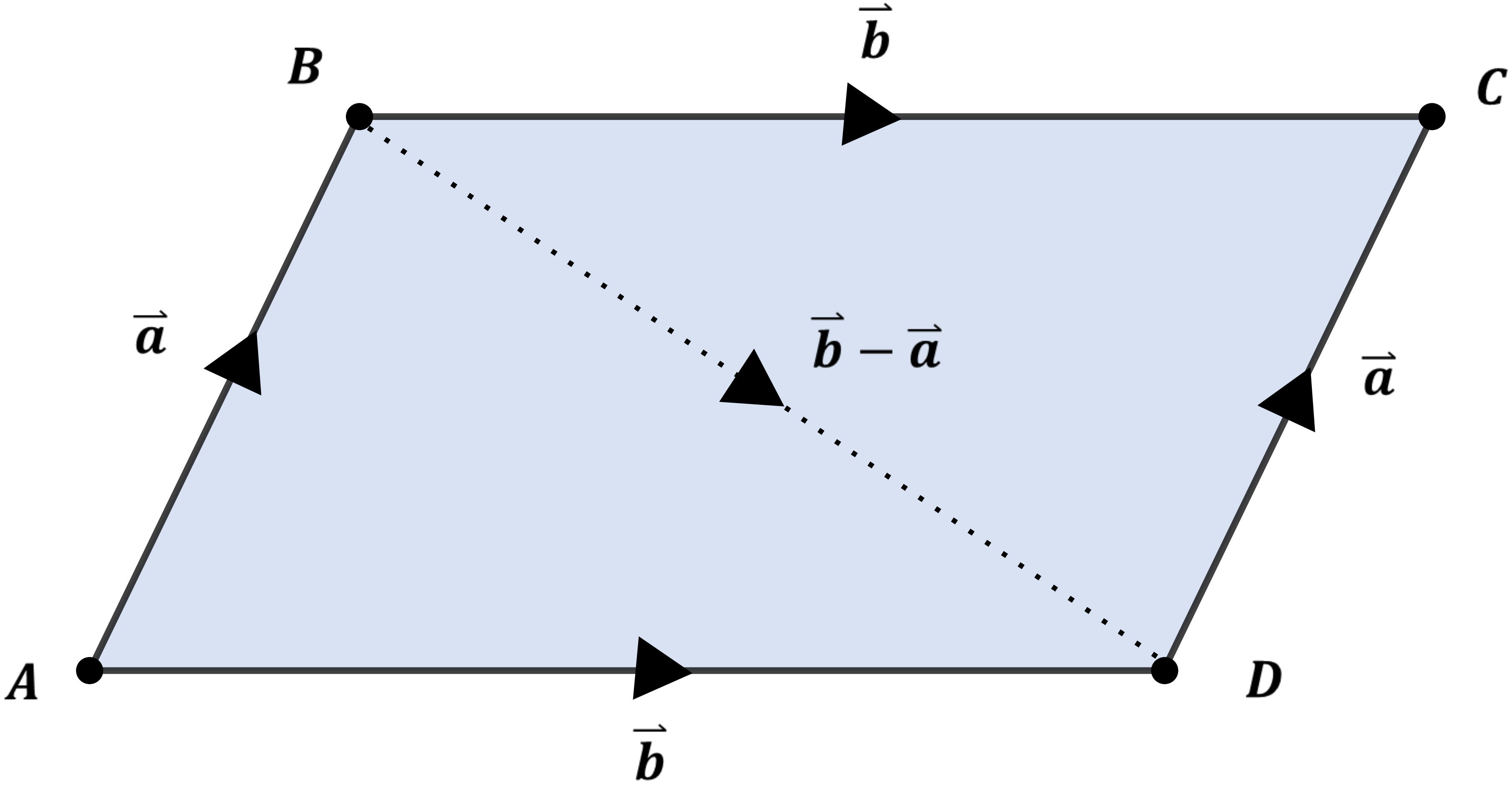
Kinematics

1. Kinematic Graphs
2. Constant Acceleration
3. Projectiles

Forces

1. Dynamics and Equilibrium

Vectors



Specification Points - AQA

	Content
J1	Use vectors in two dimensions and in three dimensions.

	Content
J2	Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.

	Content
J3	Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.

	Content
J4	Understand and use position vectors; calculate the distance between two points represented by position vectors.

	Content
J5	Use vectors to solve problems in pure mathematics and in context, including forces and kinematics.

Specification Points – OCR A

1.10 Vectors			
1.10a 1.10b	Vectors	a) Be able to use vectors in two dimensions. *i.e. Learners should be able to use vectors expressed as* $x\mathbf{i}+y\mathbf{j}$ *or as a column vector* $\begin{pmatrix}x\\y\end{pmatrix}$, *to use vector notation appropriately either as* $\overrightarrow{AB}$ *or* $\mathbf{a}$. *Learners should know the difference between a scalar and a vector, and should distinguish between them carefully when writing by hand.*	b) Be able to use vectors in three dimensions. *i.e. Learners should be able to use vectors expressed as* $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ *or as a column vector* $\begin{pmatrix}x\\y\\z\end{pmatrix}$. *Includes extending 1.10c to 1.10g to include vectors in three dimensions, excluding the direction of a vector in three dimensions.*
1.10c	Magnitude and direction of vectors	c) Be able to calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form. *Learners should know that the modulus of a vector is its magnitude and the direction of a vector is given by the angle the vector makes with a horizontal line parallel to the positive x-axis. The direction of a vector will be taken to be in the interval* $[0°, 360°)$. *Includes use of the notation* $\lvert\mathbf{a}\rvert$ *for the magnitude of* $\mathbf{a}$ *and* $\lvert\overrightarrow{OA}\rvert$ *for the magnitude of* $\overrightarrow{OA}$. *Learners should be able to calculate the magnitude of a vector* $\begin{pmatrix}x\\y\end{pmatrix}$ *as* $\sqrt{x^2+y^2}$ *and its direction by using* $\tan^{-1}\left(\frac{x}{y}\right)$.	

1.10d	Basic operations on vectors	d) Be able to add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations. *i.e. Either a scaling of a single vector or a displacement from one position to another by adding one or more vectors, often in the form of a triangle of vectors.*	
1.10e	Position vectors	e) Understand and be able to use position vectors. *Learners should understand the meaning of displacement vector, component vector, resultant vector, parallel vector, equal vector and unit vector.*	
1.10f	Distance between points	f) Be able to calculate the distance between two points represented by position vectors. *i.e. The distance between the points* $a\mathbf{i}+b\mathbf{j}$ *and* $c\mathbf{i}+d\mathbf{j}$ *is* $\sqrt{(c-a)^2+(d-b)^2}$.	
1.10g 1.10h	Problem solving using vectors	g) Be able to use vectors to solve problems in pure mathematics and in context, including forces.	h) Be able to use vectors to solve problems in kinematics. *e.g. The equations of uniform acceleration may be used in vector form to find an unknown. See section 3.02e.*

Specification Points – OCR MEI

PURE MATHEMATICS: VECTORS (1)				
General vectors	Mv1	Understand the language of vectors in two dimensions.	Scalar, vector, modulus, magnitude, direction, position vector, unit vector, cartesian components, equal vectors, parallel vectors, collinear.	Vectors printed in **bold**. Unit vectors $\mathbf{i}$, $\mathbf{j}$, $\hat{\mathbf{r}}$ The magnitude of the vector $\mathbf{a}$ is written $\lvert\mathbf{a}\rvert$ or a. $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$
	v2	Be able to add and subtract vectors using a diagram or algebraically, multiply a vector by a scalar, and express a vector as a combination of others.	Geometrical interpretation. Includes general vectors not expressed in component form.	
	v3	Be able to calculate the magnitude and direction of a vector and convert between component form and magnitude-direction form.		Magnitude-direction
Position vectors	v4	Understand and use position vectors.	Including interpreting components of a position vector as the cartesian coordinates of the point. $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$	$\overrightarrow{OB}$ or $\mathbf{b}$. $\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}$
	v5	Be able to calculate the distance between two points represented by position vectors.		
Using vectors	v6	Be able to use vectors to solve problems in pure mathematics and in context, including problems involving forces.	Includes interpreting the sum of vectors representing forces as the resultant force.	
PURE MATHEMATICS: VECTORS (2)				
General vectors	Mv7	Understand the language of vectors in three dimensions.	Extend the work of Mv2 to Mv 6 to include vectors in three dimensions.	Unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, $\hat{\mathbf{r}}$ $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$

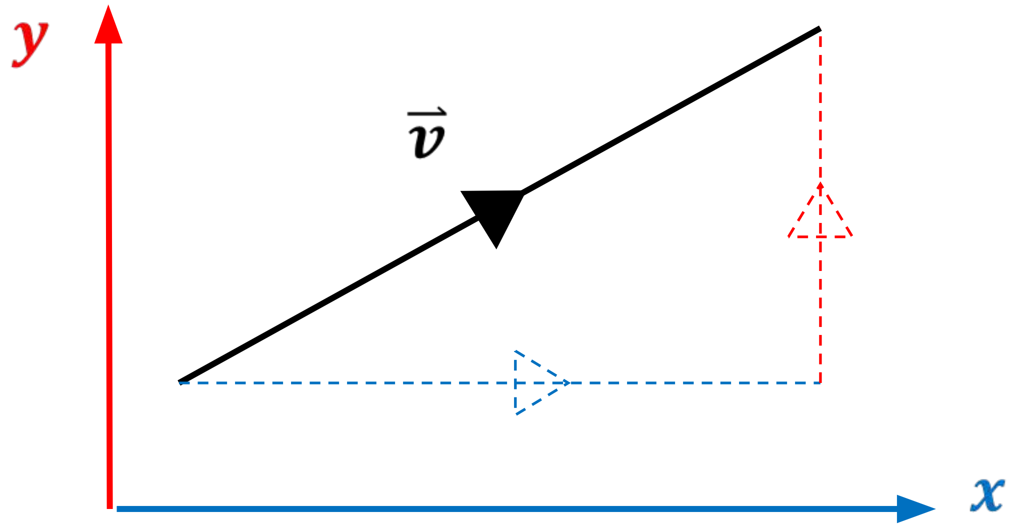
Specification Points - Edexcel

10.1	**Use vectors in two dimensions** and in three dimensions	**Students should be familiar with column vectors and with the use of i and j unit vectors in two dimensions** and **i**, **j** and **k** unit vectors in three dimensions.
10.2	**Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.**	**Students should be able to find a unit vector in the direction of a, and be familiar with the notation $\lvert a \rvert$.**
10.3	**Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.**	**The triangle and parallelogram laws of addition.** **Parallel vectors.**

10.4	**Understand and use position vectors; calculate the distance between two points represented by position vectors.**	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ **The distance d between two points (x_1, y_1) and (x_2, y_2) is given by** $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$
10.5	**Use vectors to solve problems in pure mathematics and in context, (including forces).**	**For example, finding position vector of the fourth corner of a shape (e.g. parallelogram) $ABCD$ with three given position vectors for the corners A, B and C.** **Or use of ratio theorem to find position vector of a point C dividing AB in a given ratio.** **Contexts such as velocity, displacement, kinematics and forces will be covered in Paper 3, Sections 6.1, 7.3 and 8.1 – 8.4**

Vector Notation

A **vector** is a **quantity** with both a specified **magnitude** (**size**) and specified **direction**.

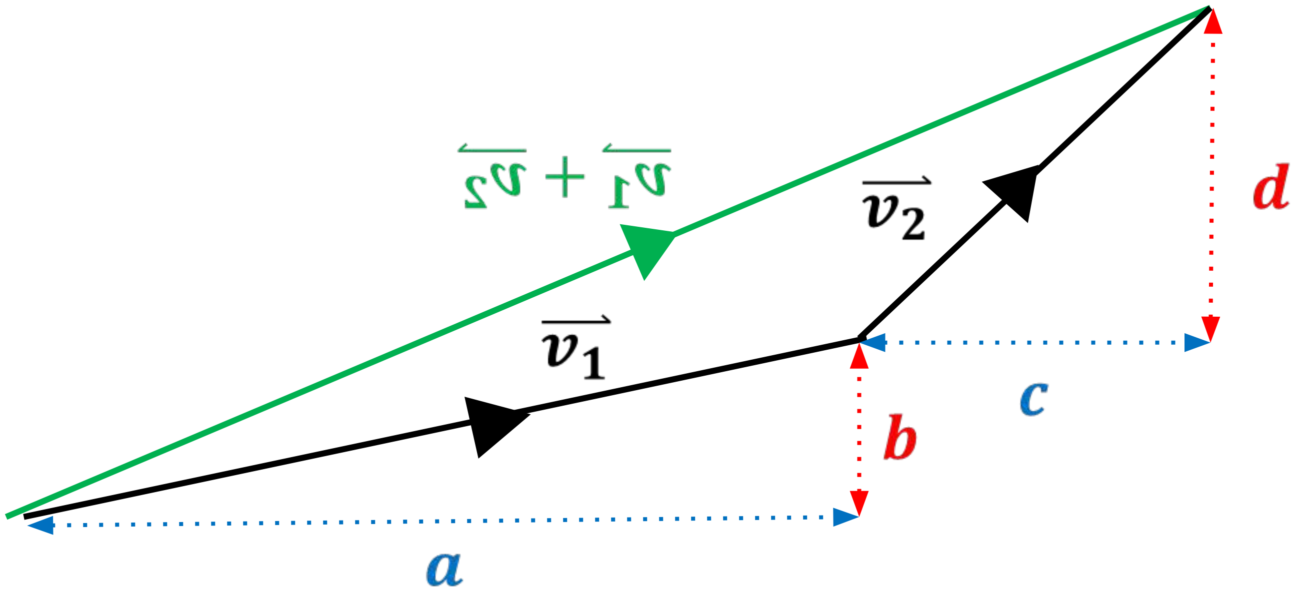


Any $\mathbf{2.D}$ **vector** can be viewed as the **sum** of $\mathbf{2}$ **perpendicular vectors** in the x and y **directions**.

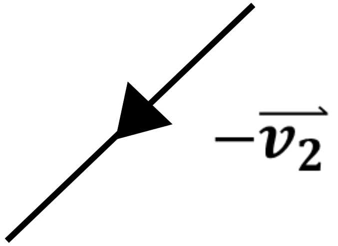
- The **magnitude** of a **vector** is **written** $|\vec{v}|$
- A **unit vector** is a **vector** with **magnitude** $= \mathbf{1}$:

Vector Notation

Combinations of any **2 non-parallel vectors** can **define** any **point** on a **2. D plane**.
The **addition** of 2 or more **vectors** is known as a **resultant vector**.



$= \overrightarrow{v_1} + \overrightarrow{v_2}$



$$\overrightarrow{v_1} = \begin{pmatrix} a \\ b \end{pmatrix} = a\hat{\imath} + b\hat{\jmath} \qquad \overrightarrow{v_2} = \begin{pmatrix} c \\ d \end{pmatrix} = c\hat{\imath} + d\hat{\jmath}$$

Exemplar Exam Question

1) Consider the vectors $\overrightarrow{v_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\overrightarrow{v_2} = \begin{pmatrix} -6 \\ a \end{pmatrix}$. Their resultant vector $\vec{v}$ has magnitude $= 5$.

(i) Determine the possible values of a

More than one value, **expect a quadratic**

(ii) State the magnitude of the vector $\overrightarrow{v_3}$ such that $\overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3} = 0$

[4 marks]

State implies **no working required**, this is a **question** about **understanding** of **vectors** rather than **calculation**

4 steps in total, between two questions

Exemplar Exam Question Answer

$$\overrightarrow{v_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad \overrightarrow{v_2} = \begin{pmatrix} -6 \\ a \end{pmatrix}$$

(i) Calculate resultant vector $\vec{v}$

For $\vec{u}_1 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}, \vec{u}_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$, resultant vector $\vec{u}_1 + \vec{u}_2 = \begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \end{pmatrix}$

$$\vec{v} = \overrightarrow{v_1} + \overrightarrow{v_2}$$

$$\vec{v} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} -6 \\ a \end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} 2 - 6 \\ 4 + a \end{pmatrix} = \begin{pmatrix} -4 \\ 4 + a \end{pmatrix}$$

Exemplar Exam Question Answer

$\vec{v_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $\vec{v_2} = \begin{pmatrix} -6 \\ a \end{pmatrix}$ $\vec{v} = \begin{pmatrix} -4 \\ 4 + a \end{pmatrix}$

Calculate magnitude of $\vec{v}$

Magnitude of vector $\vec{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ is given by $|\vec{u}| = \sqrt{a^2 + b^2}$

$$|\vec{v}| = \sqrt{(-4)^2 + (4 + a)^2}$$

$$= \sqrt{16 + 16 + 8a + a^2}$$

$$= \sqrt{32 + 8a + a^2}$$

[1 Mark]

Exemplar Exam Question Answer

$$\vec{v_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad \vec{v_2} = \begin{pmatrix} -6 \\ a \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} -4 \\ 4 + a \end{pmatrix}$$

Use magnitude of $\vec{v}$ to calculate a

Know that magnitude of $\vec{v}$ is 5

$$|\vec{v}| = \sqrt{32 + 8a + a^2}$$

$$5 = \sqrt{32 + 8a + a^2}$$

[1 Mark]

Square both sides

$$25 = 32 + 8a + a^2$$

Exemplar Exam Question Answer

$\vec{v_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $\vec{v_2} = \begin{pmatrix} -6 \\ a \end{pmatrix}$ $\vec{v} = \begin{pmatrix} -4 \\ 4 + a \end{pmatrix}$

Bring all terms to one side

$$25 = 32 + 8a + a^2$$

$$a^2 + 8a + 7 = 0$$

Factorise

$$0 = (a + 1)(a + 7)$$

So $a = -1$ or -7

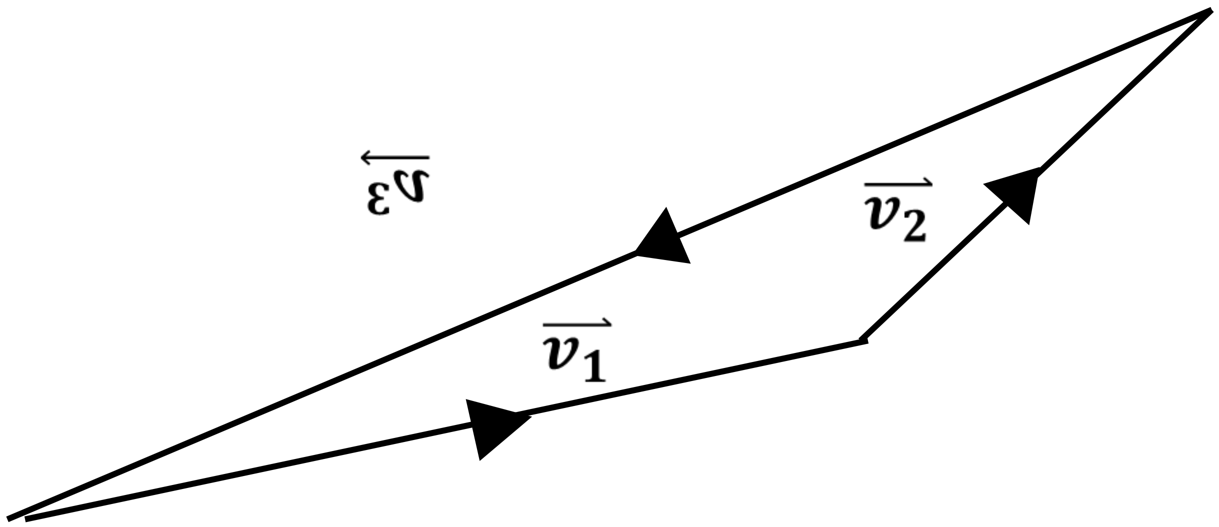
[1 Mark]

Exemplar Exam Question Answer

$\overrightarrow{v_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $\overrightarrow{v_2} = \begin{pmatrix} -6 \\ a \end{pmatrix}$ $\vec{v} = \begin{pmatrix} -4 \\ 4 + a \end{pmatrix}$

(ii) Deduce value of $|\overrightarrow{v_3}|$

Consider diagram.



Exemplar Exam Question Answer

$\overrightarrow{v_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $\overrightarrow{v_2} = \begin{pmatrix} -6 \\ a \end{pmatrix}$ $\vec{v} = \begin{pmatrix} -4 \\ 4 + a \end{pmatrix}$

For resulting vector $\overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3}$ to be $\mathbf{0}$, need result to be at origin

So $\overrightarrow{v_3} = -\vec{v}$

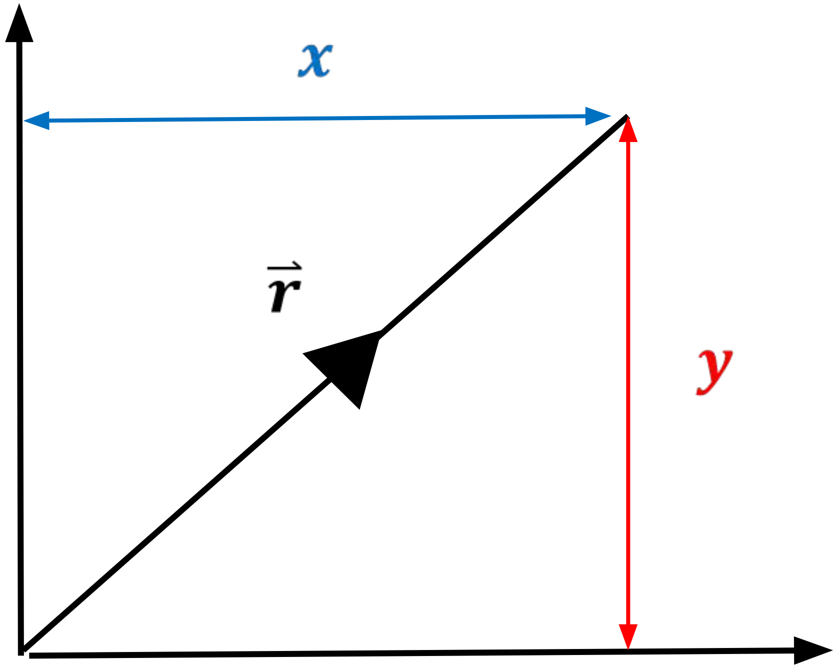
$\overrightarrow{v_3}$ and $\vec{v}$ have same length

So $|\overrightarrow{v_3}| = |\vec{v}| = 5$

[1 Mark]

Resolving Vectors

We can **resolve a vector** by taking its **components** in **perpendicular** x (**horizontal**) and y (**vertical**) **directions**.



Converting from one **form** to another

Exemplar Exam Question

1) Consider the vector $\vec{v}$, which has magnitude $4\sqrt{3}$ and makes an angle of 30° with the x-axis.

(i) Write $\vec{v}$ in the form $a\hat{\imath} + b\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors in the x and y directions respectively.

(ii) Let $\vec{w} = \vec{v} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Using your previous result, calculate the angle $\vec{w}$ makes with the horizontal, giving your answer to 3 significant figures.

Remember to **round**

3 marks, 1 for one part, **2** for the other

[3 marks]

Exemplar Exam Question

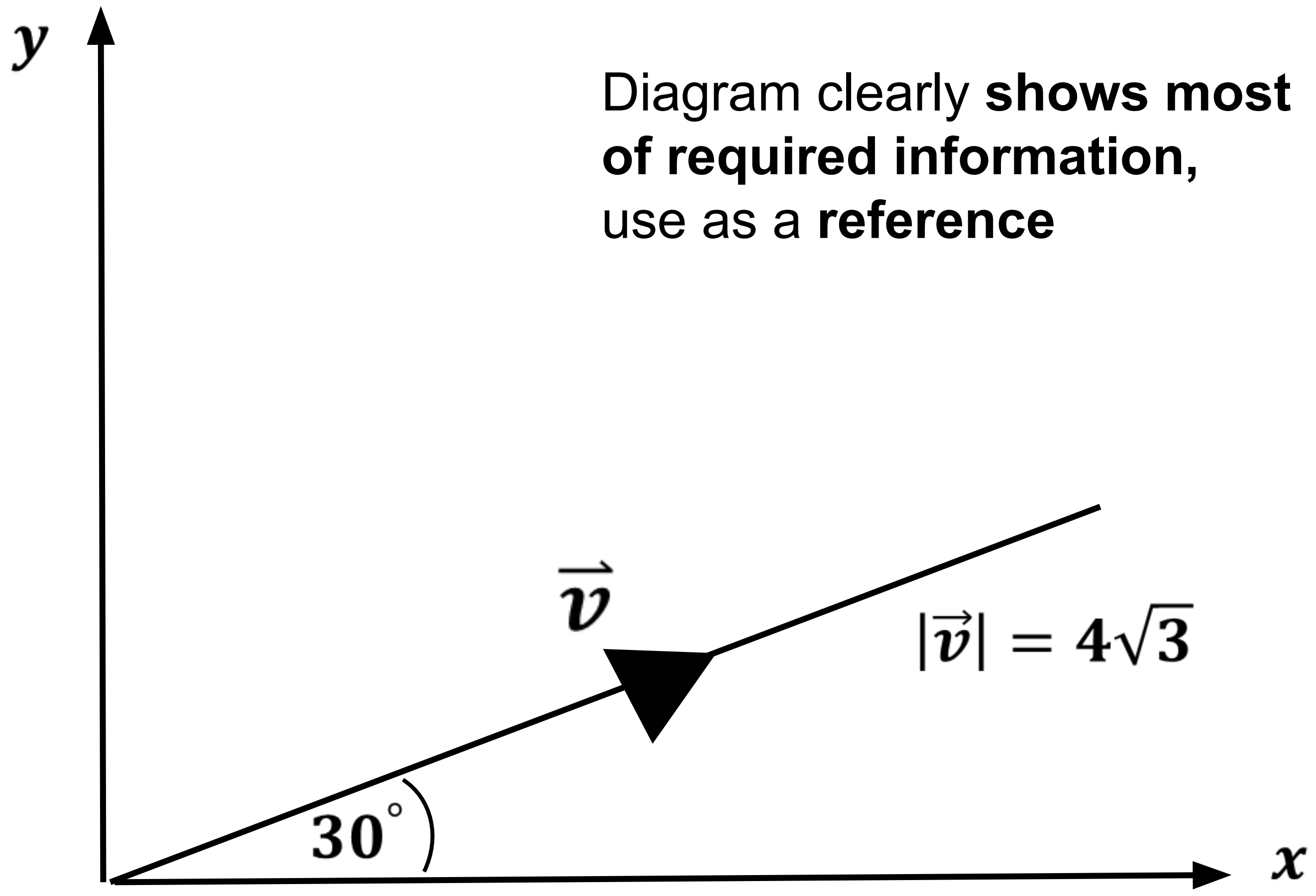


Diagram clearly **shows most of required information,** use as a **reference**

Exemplar Exam Question Answer

(i) Calculate components using given information.

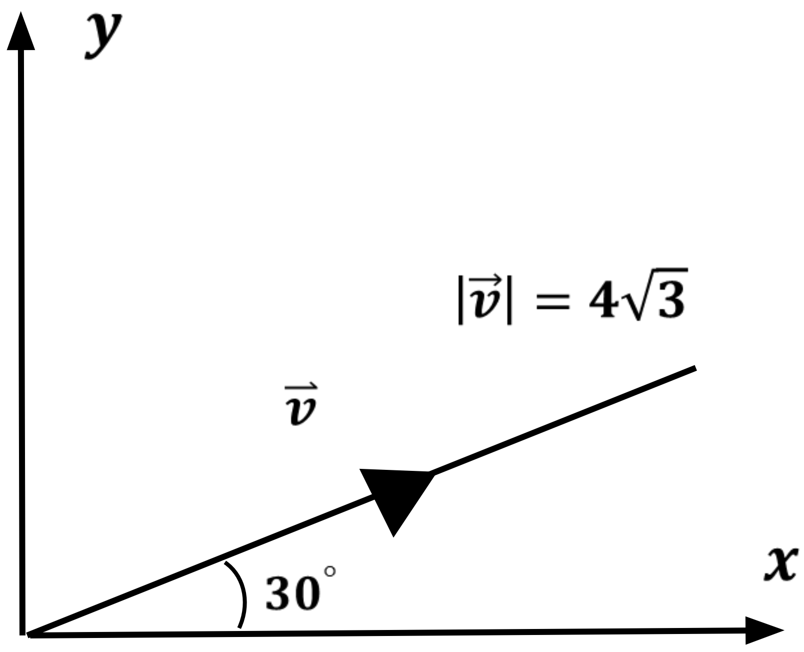
Know vector $\vec{r}$ can be resolved as

$$\vec{r} = |\vec{r}|\cos\theta\,\hat{\imath} + |\vec{r}|\sin\theta\,\hat{\jmath}$$

From $\vec{v} = a\hat{\imath} + b\hat{\jmath}$, deduce

$$a = |\vec{v}|\cos\theta$$

$$b = |\vec{v}|\sin\theta$$



Exemplar Exam Question Answer

Substitute $|\vec{v}| = 4\sqrt{3}$ and $\theta = 30°$

$a = |\vec{v}|\cos\theta$

$a = 4\sqrt{3}\cos 30$

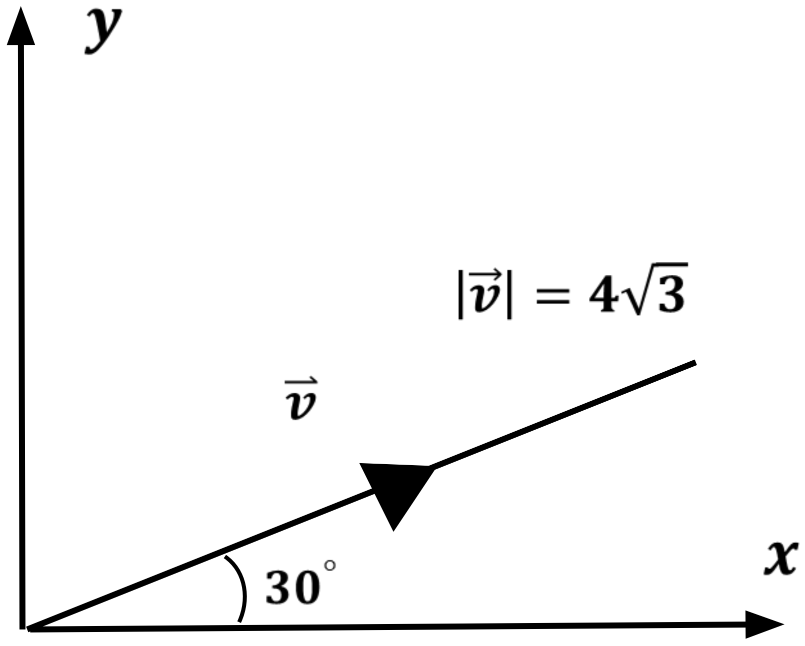
$a = 6$

$b = |\vec{v}|\sin\theta$

$b = 4\sqrt{3}\sin 30$

$b = 2\sqrt{3}$

$\vec{v} = 6\hat{\imath} + 2\sqrt{3}\hat{\jmath}$



[1 Mark]

Exemplar Exam Question Answer

(ii) Calculate new vector $\vec{w}$

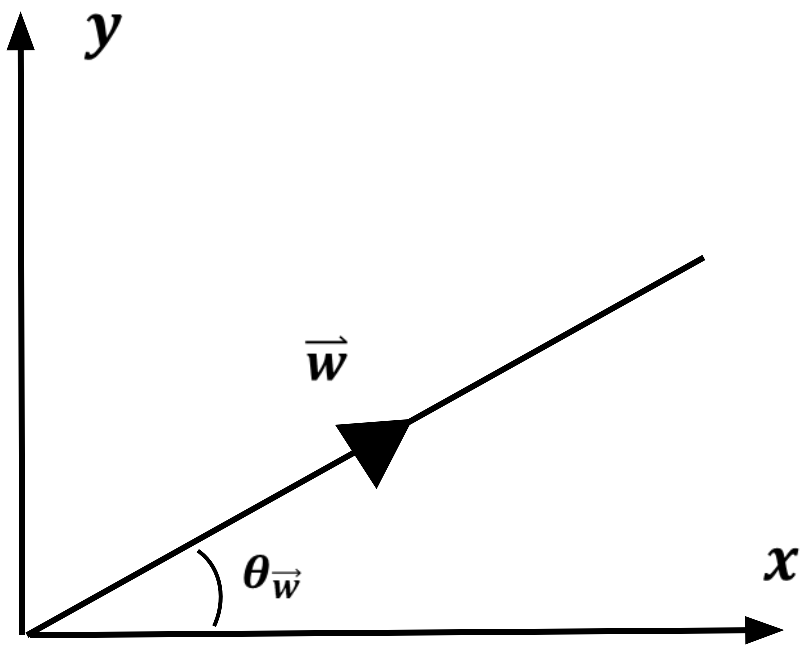
Find resultant vector by addition

$$\vec{w} = \vec{v} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} 6 \\ 2\sqrt{3} \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} 7 \\ 2\sqrt{3} + 1 \end{pmatrix}$$

[1 Mark]



Exemplar Exam Question Answer

Calculate angle with x-axis

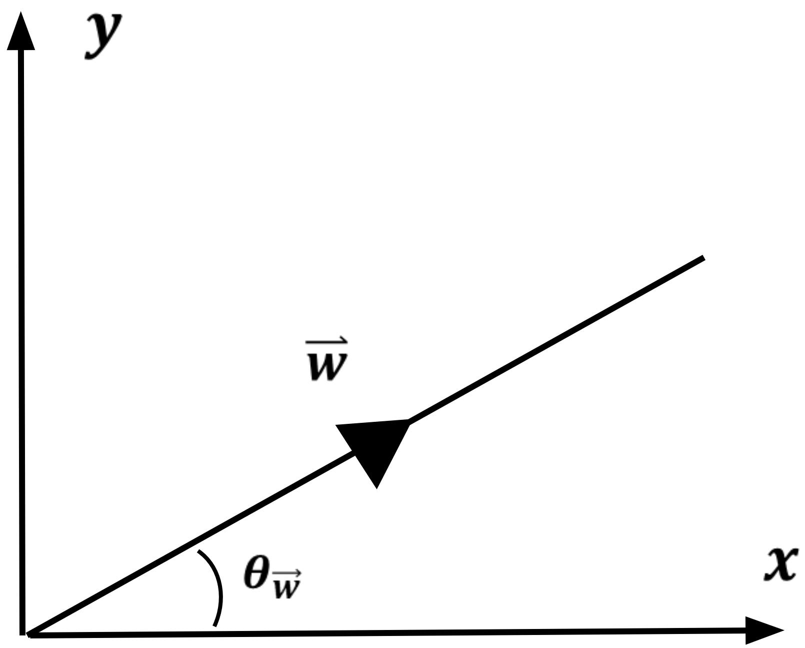
For vector $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath}$, angle with x-axis given by

$$\tan\theta = \frac{y}{x}$$

Apply formula to $\vec{w}$

$$\vec{w} = \begin{pmatrix} 7 \\ 2\sqrt{3}+1 \end{pmatrix}$$

$$\tan\theta_{\vec{w}} = \frac{2\sqrt{3}+1}{7}$$



Exemplar Exam Question Answer

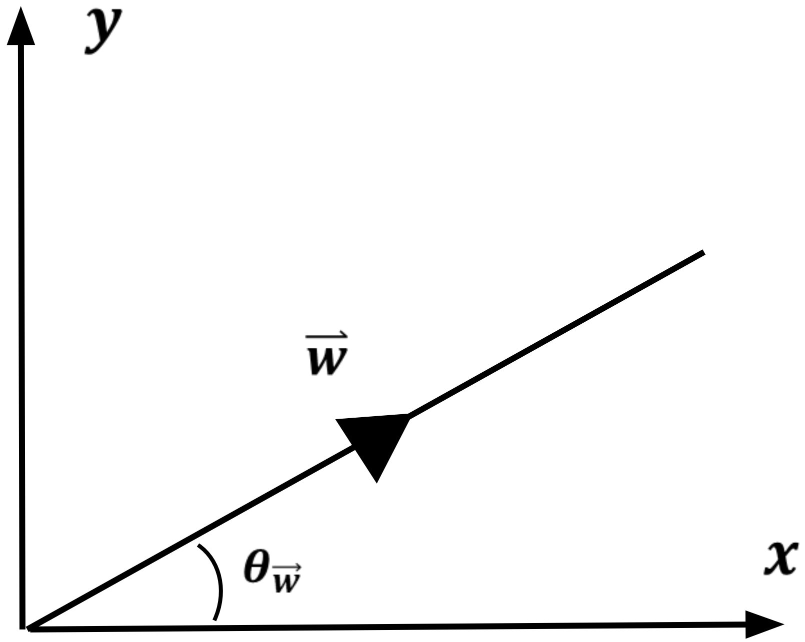
Calculate angle with x-axis

$$\tan\theta_{\vec{w}} = \frac{2\sqrt{3}+1}{7}$$

$$\theta_{\vec{w}} = \tan^{-1}\left(\frac{2\sqrt{3}+1}{7}\right)$$

$$\theta_{\vec{w}} = 32.5°$$

[1 Mark]



Kinematics

Specification Points - AQA

	Content
Q1	Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.

	Content
Q2	Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph.

	Content
Q3	Understand, use and derive the formulae for constant acceleration for motion in a straight line; extend to 2 dimensions using vectors.

	Content
Q5	Model motion under gravity in a vertical plane using vectors; projectiles.

Specification Points – OCR A

3.02 Kinematics		
3.02a	Language of kinematics	a) Understand and be able to use the language of kinematics: position, displacement, distance, distance travelled, velocity, speed, acceleration, equation of motion. *Learners should understand the vector nature of displacement, velocity and acceleration and the scalar nature of distance travelled and speed.*
3.02b 3.02c	Graphical representation	b) Understand, use and interpret graphs in kinematics for motion in a straight line. c) Be able to interpret displacement-time and velocity-time graphs, and in particular understand and be able to use the facts that the gradient of a displacement-time graph represents the velocity, the gradient of a velocity-time graph represents the acceleration, and the area between the graph and the time axis for a velocity-time graph represents the displacement.

3.02d 3.02e	Constant acceleration	d) Understand, use and derive the formulae for constant acceleration for motion in a straight line: $v = u + at$ $s = ut + \frac{1}{2}at^2$ $s = \frac{1}{2}(u + v)t$ $v^2 = u^2 + 2as$ $s = vt - \frac{1}{2}at^2$ *Learners may be required to derive the constant acceleration formulae using a variety of techniques:* 1. *by integration, e.g.* $v = \int a\,dt \Rightarrow v = u + at$, 2. *by using and interpreting appropriate graphs, e.g. velocity against time,* 3. *by substitution of one (given) formula into another (given) formula, e.g. substituting* $v = u + at$ *into* $s = \frac{1}{2}(u + v)t$ *to obtain* $s = ut + \frac{1}{2}at^2$.	e) Be able to extend the constant acceleration formulae to motion in two dimensions using vectors: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ *Questions set involving vectors may involve either column vector notation, e.g.* $\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ *or* **i**, **j** *notation, e.g.* $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$. *[The formula* $\mathbf{v} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{u} + 2\mathbf{a} \cdot \mathbf{s}$ *is excluded.]*

Specification Points – OCR A

3.02h 3.02i	Gravity	h) Be able to model motion under gravity in a vertical plane using vectors where $\mathbf{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$ or $\mathbf{a} = -g\mathbf{j}$. i) Be able to model the motion of a projectile as a particle moving with constant acceleration and understand the limitation of this model. *Includes being able to:* *1. Use horizontal and vertical equations of motion to solve problems on the motion of projectiles.* *2. Find the magnitude and direction of the velocity at a given time or position.* *3. Find the range on a horizontal plane and the greatest height achieved.* *4. Derive and use the cartesian equation of the trajectory of a projectile.* *[Projectiles on an inclined plane and problems with resistive forces are excluded.]*

Specification Points – OCR MEI

Mk1	Understand and use the language of kinematics.	Position, displacement, distance travelled; speed, velocity; acceleration, magnitude of acceleration; relative velocity (in 1-dimension). Average speed = distance travelled ÷ elapsed time Average velocity = overall displacement ÷ elapsed time
k2	Know the difference between position, displacement, distance and distance travelled.	
k3	Know the difference between velocity and speed, and between acceleration and magnitude of acceleration.	
k4	Be able to draw and interpret kinematics graphs for motion in a straight line, knowing the significance (where appropriate) of their gradients and the areas underneath them.	Position-time, displacement-time, distance-time, velocity-time, speed-time, acceleration-time.
k7	Be able to recognise when the use of constant acceleration formulae is appropriate.	Learners should be able to derive the formulae.
k8	Be able to solve kinematics problems using constant acceleration formulae and calculus for motion in a straight line.	

My1	Be able to model motion under gravity in a vertical plane using vectors. Be able to formulate the equations of motion of a projectile using vectors.	Standard modelling assumptions for projectile motion are as follows. • No air resistance. • The projectile is a particle. • Horizontal distance travelled is small enough to assume that gravity is always in the same direction. • Vertical distance travelled is small enough to assume that gravity is constant.
y2	Know how to find the position and velocity at any time of a projectile and find range and maximum height.	
y3	Be able to find the initial velocity of a projectile given sufficient information.	
y4	Be able to eliminate time from the component equations that give the horizontal and vertical displacement in terms of time to obtain the equation of the trajectory.	
y5	Be able to solve simple problems involving projectiles.	

Mk9	Understand the language of kinematics appropriate to motion in 2 dimensions. Know the difference between, displacement, distance from and distance travelled; velocity and speed, and between acceleration and magnitude of acceleration.	Position vector, relative position. Average speed = distance travelled ÷ elapsed time Average velocity = overall displacement ÷ elapsed time
k10	Be able to extend the scope of techniques from motion in 1 dimension to that in 2 dimensions by using vectors.	The use of calculus and the use of constant acceleration formulae.
k11	Be able to find the cartesian equation of the path of a particle when the components of its position vector are given in terms of time.	
k12	Be able to use vectors to solve problems in kinematics.	Includes relative position of one particle from another. Includes knowing that the velocity vector gives the direction of motion and the acceleration vector gives the direction of resultant force.

Specification Points - Edexcel

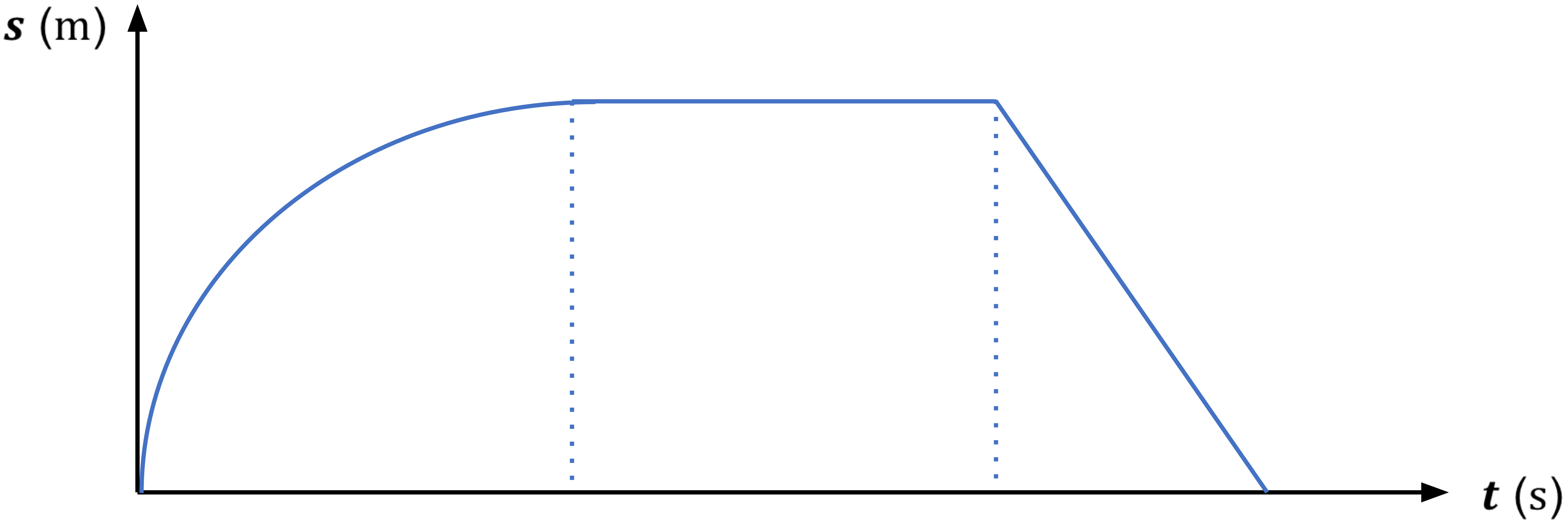
7.1	**Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.**	Students should know that distance and speed must be positive.
7.2	**Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph.**	**Graphical solutions to problems may be required.**
7.3	**Understand, use and derive the formulae for constant acceleration for motion in a straight line.** Extend to 2 dimensions using vectors.	**Derivation may use knowledge of sections 7.2 and/or 7.4** **Understand and use *suvat* formulae for constant acceleration in 2-D,** e.g. $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with vectors given in $\mathbf{i} - \mathbf{j}$ or column vector form. Use vectors to solve problems.

7.5	Model motion under gravity in a vertical plane using vectors; projectiles.	Derivation of formulae for time of flight, range and greatest height and the derivation of the equation of the path of a projectile may be required.

Kinematic Graphs

Displacement-time graphs plot an object's **displacement** against **time**.

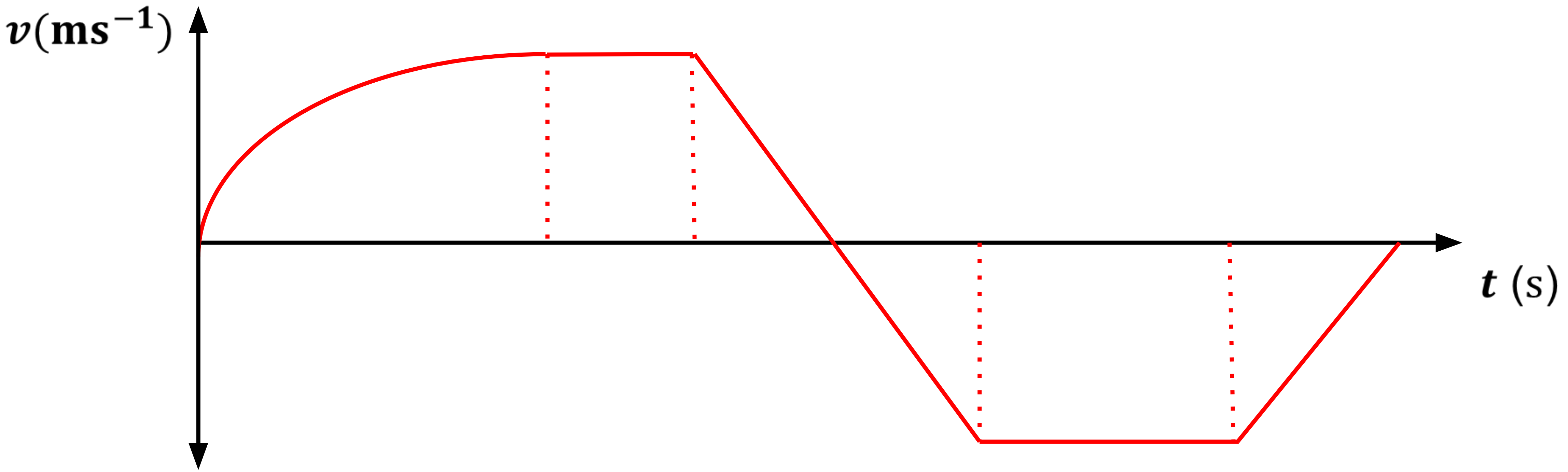
- The **gradient** of the **graph** gives the **instantaneous velocity** at that **time**.



Kinematic Graphs

Velocity-time graphs plot an object's **velocity** against **time**.

- The **gradient** of the **graph** gives the **instantaneous acceleration** at that **time**.
- The **area** between the **graph** and the **time-axis** gives the **total distance travelled**.



Check details to fully understand info on graph

Exemplar Exam Question

1) Elliott typically cycles to work. He starts his day at home and works in his office from 9am to 5pm. His journey is shown on the following time-displacement graph, where s is in km and t is in hours after 7am.

Questions on **interpreting graphs**

(i) Determine how far Elliott's office is from his home.

(ii) Calculate Elliot's speed on his way home, giving your answer in an appropriate unit.

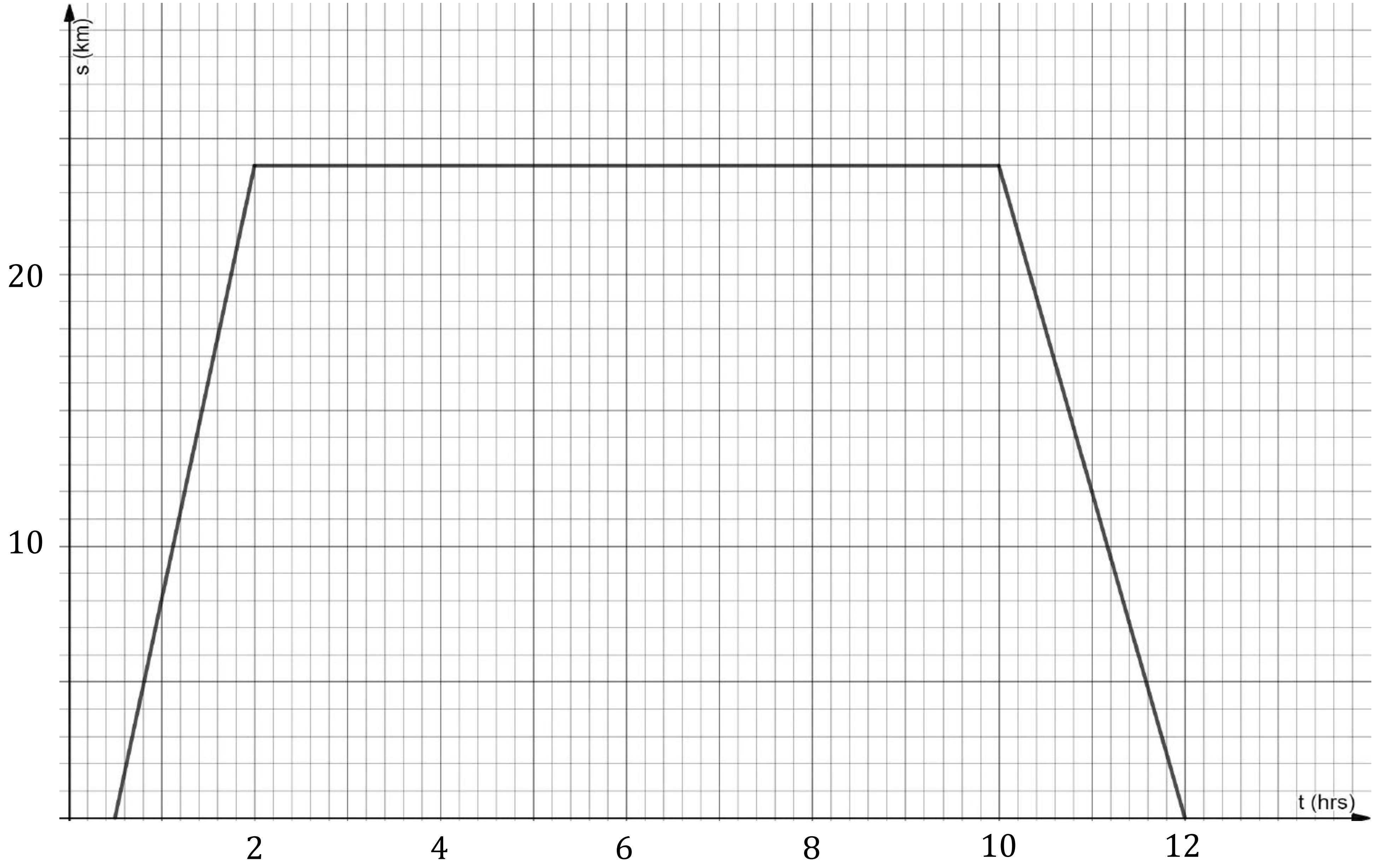
Elliott receives a new bike for Christmas, which lets him cycle twice as fast to and from work. He still works the same hours.

(iii) Add new lines to the displacement-time graph to show how his journey has changed, clearly showing the new times he leaves and returns home.

[4 marks]

Need **precision** for the new lines, **calculate** new values

4 steps, first two parts are **quick**, last takes a **bit longer**


s (km)
20
10
t (hrs)
2
4
6
8
10
12

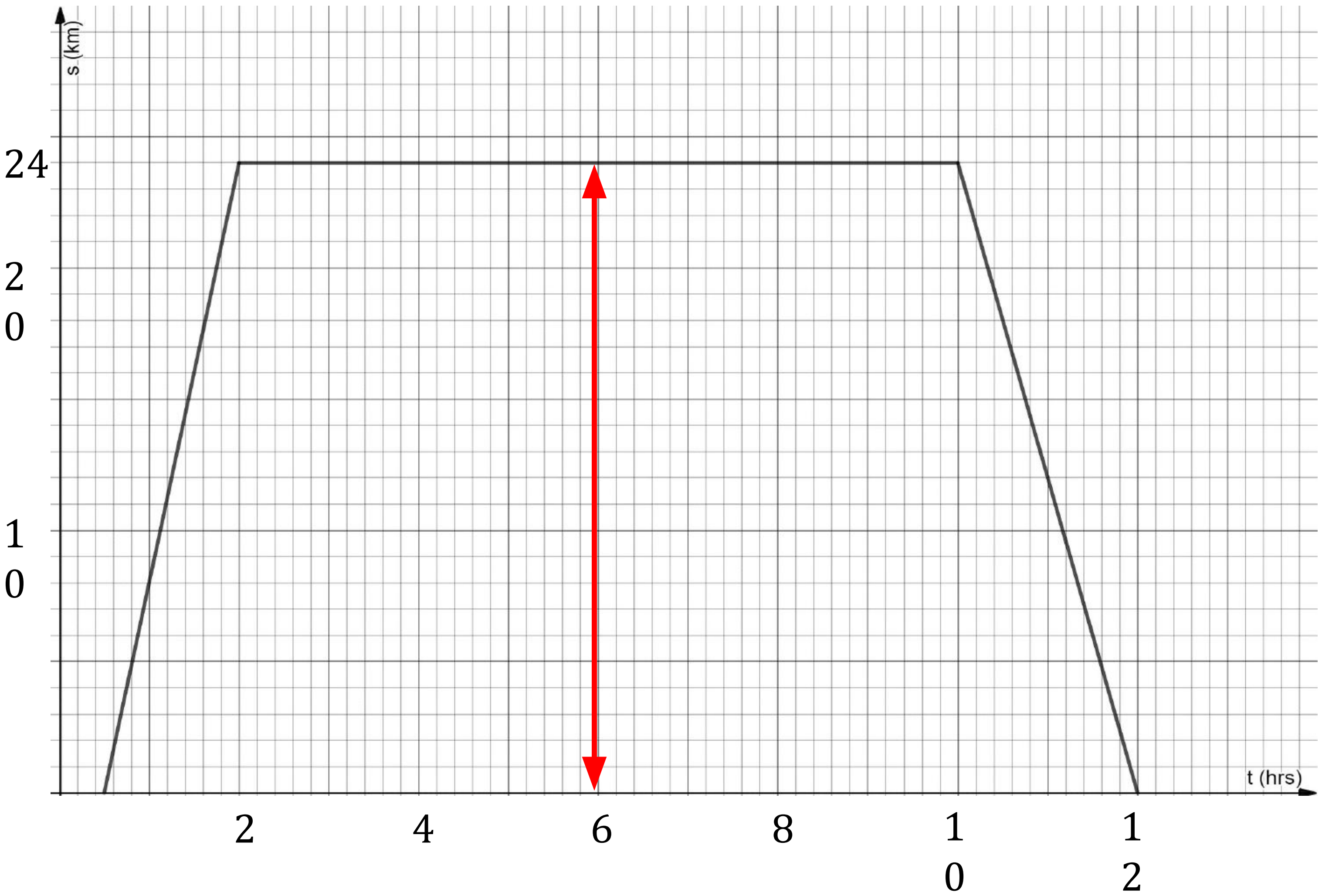
Exemplar Exam Question Answer

(i) Find displacement from starting point when Elliott is at work

Elliott is at work from 9 to 5, or the flat line part of the graph

Can simply read off s value from graph

Exemplar Exam Question Answer



Distance is
24 km

[1 Mark]

Exemplar Exam Question Answer

(ii) Calculate gradient for final part of graph to determine speed of return journey.

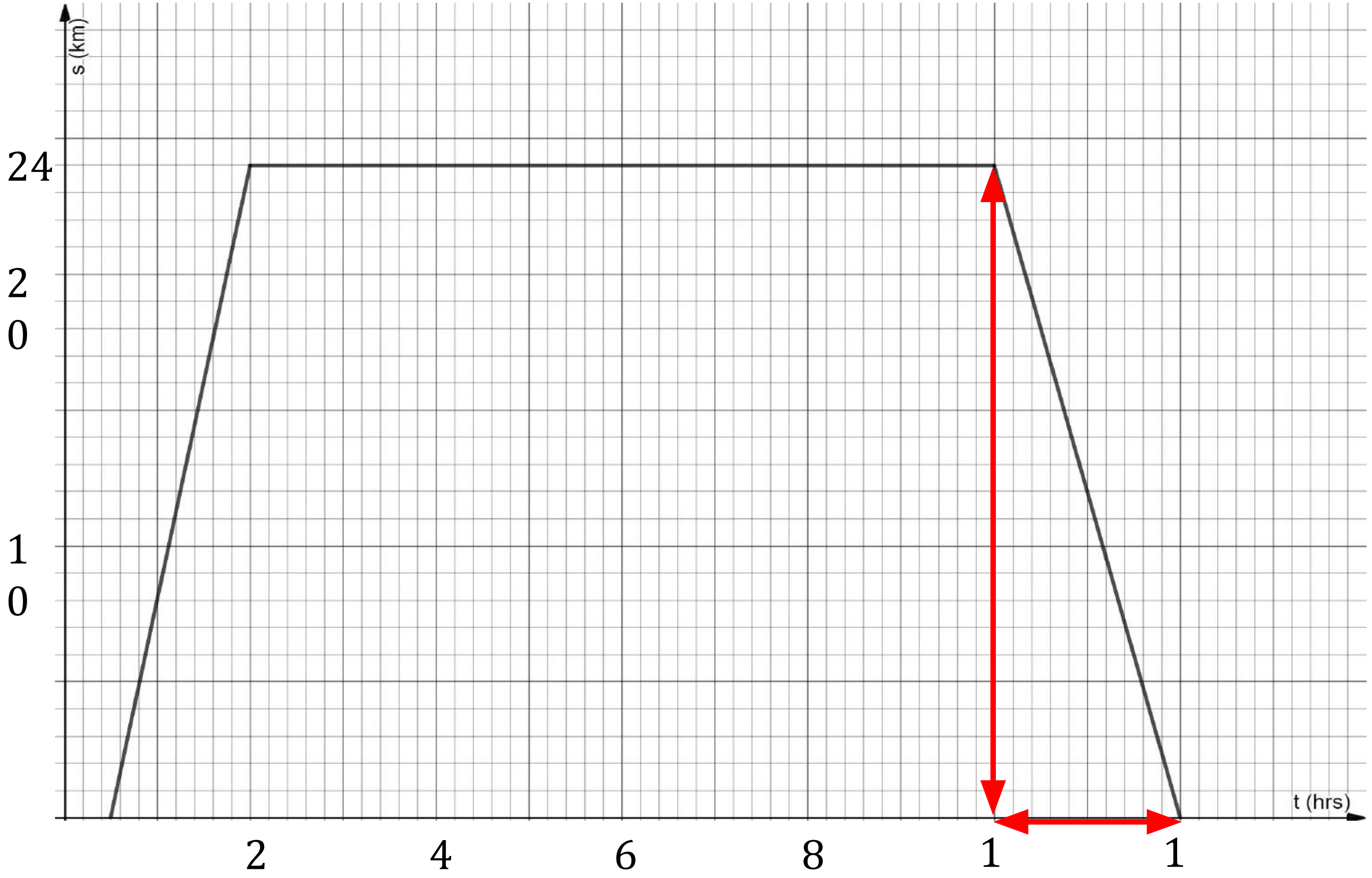
Velocity is given by gradient of a displacement-time graph.

Speed is magnitude of velocity.

Elliot's journey home is given by final part of graph, so need to calculate gradient for this part.

Use units given in graph. We're dividing distance by time so use km/h.

Exemplar Exam Question Answer



Gradient is given by $\frac{\Delta y}{\Delta x}$

So speed is given by

$$\frac{24-0}{12-10}=\frac{24}{2}$$

$$=12\text{kmh}^{-1}$$

[1 Mark]

Exemplar Exam Question Answer

(iii) Determine start and end points of new lines for journey at twice previous speed.

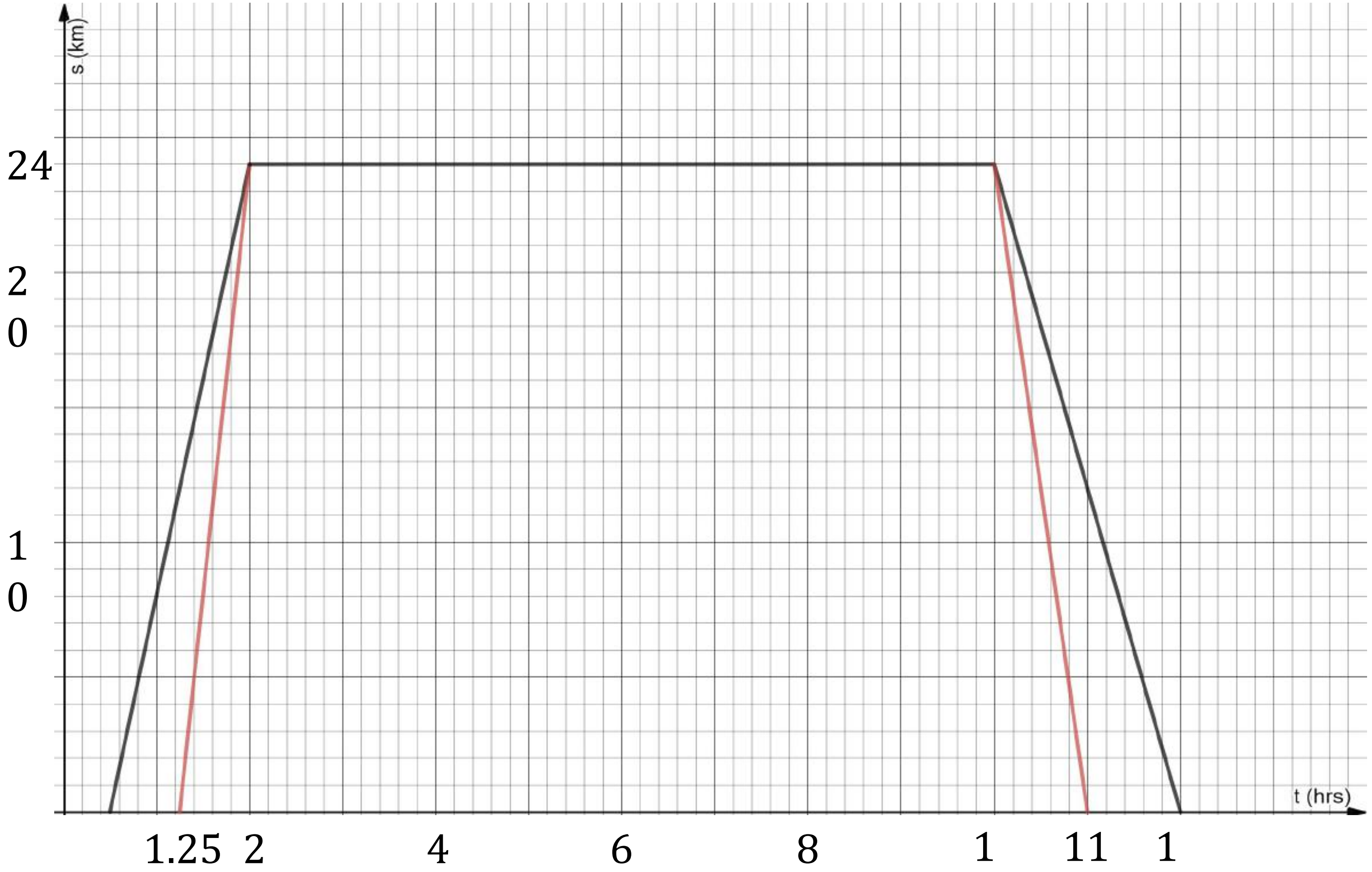
Elliot's to-work and from-work speed will double.

First line will still end at $t = 2$, final line will still start at $t = 10$.

Gradient will be twice as steep.

[1 Mark]

Exemplar Exam Question Answer



[1 Mark]

Constant Acceleration

The **SUVAT equations** for **constant acceleration** relate the **5 variables**:

- s - **Displacement**
- u – **Initial Velocity**
- v – **Final Velocity**
- a – **Acceleration**
- t – **Time**

$$v = u + at$$

$$s = \frac{(u+v)}{2}t$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

Each **equation** contains 4 of the 5 **variables**. If you know 3 of the **variables** you can use the right **SUVAT equations** to find the others.

Constant Acceleration

Write down the **variables** you **know** and the **variable** you need to **calculate**. Then **select** the **equation** which contains these **variables**.

- If the **object** begins **at rest**, $u = 0$. This can **simplify** some of the **equations**.

$$s = ut + \frac{1}{2}at^2 \qquad\qquad v^2 = u^2 + 2as$$

- For **motion** under **gravity**, the **acceleration** a **equals** a **constant** $g = 9.8(1)\text{ms}^{-2}$.
- If the **object** is **thrown** up in the air, $v = 0$ at its **highest point**. Also the **time of ascent equals** the **time of descent**.

Exemplar Exam Question

Lot of **values** to plug into **SUVAT equations**

1) A coyote is trying to catch a fast, flightless bird.

The coyote runs at a constant speed of $\mathbf{10\ ms^{-1}}$. When it is $\mathbf{80\ m}$ from the bird, the bird runs away. It starts running from rest with constant acceleration.

(i) After 4s the bird is running at $\mathbf{8\ ms^{-1}}$. Calculate the bird's acceleration.
(ii) If when the bird started running the coyote starts accelerating at $\mathbf{2.5\ ms^{-2}}$, calculate the time it would take the coyote to catch the bird after the latter starts running. Give your answer to 3 significant figures

[5 marks]

2 moving bodies with **different conditions**

5 marks across **2 parts,** one requires **more work** than the other

Exemplar Exam Question Answer

(i) Write down what is known

Trying to find bird's acceleration a.

$u =$ Bird's starting speed $= 0\ \text{ms}^{-1}$

$v =$ Brid's top speed $= 8\ \text{ms}^{-1}$

$t =$ Distance of acceleration $= 4\ \text{s}$

$$\boxed{v = u + at} \qquad s = \frac{(u+v)}{2}t$$

$$v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$$

[1 Mark]

Exemplar Exam Question Answer

Rearrange *SUVAT* equation

$$u = 0$$
$$v = 8$$
$$t = 4$$

Make a subject of equation.

$$v = u + at$$

$$v - u = at$$

$$a = \frac{v-u}{t}$$

Calculate value

$$a = \frac{8-0}{4} = \frac{8}{4} = 2\ \text{ms}^{-2}$$

[1 Mark]

Exemplar Exam Question Answer

(ii) Write down what is known

Trying to find time coyote and bird meet: t.

u_c = Coyote's initial speed $= 10\ \text{ms}^{-1}$

u_b = Bird's initial speed $= 0\ \text{ms}^{-1}$

a_c = Coyote's acceleration $= 2.5\ \text{ms}^{-2}$

a_b = Bird's acceleration $= 2\ \text{ms}^{-1}$

Determine relation between bird and coyote

Bird is initially $80\ \text{m}$ from coyote and is still travelling

So coyote catches bird when it travels $80\ \text{m}$ plus the distance the bird has travelled, therefore:

$$s_c = 80\ \text{m} + \text{distance bird travels} = 80 + s_b$$

Exemplar Exam Question Answer

$u_c = 10$

$u_b = 0$

$a_c = 2.5$

$a_b = 2$

$t = ?$

$s_c = 80 + s_b$

Choose *SUVAT* equation to use

$v = u + at$

$s = \frac{(u+v)}{2}t$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

Same equation for both

[1 Mark]

Plug in values for coyote

$$s_c = u_c t + \frac{1}{2}a_c t^2$$

$$80 + s_b = 10t + \frac{1}{2}2.5t^2$$

$$80 + s_b = 10t + 1.25t^2$$

Plug in values for bird

$$s_b = u_b t + \frac{1}{2}a_b t^2$$

$$s_b = 0 + \frac{1}{2}2t^2$$

$$s_b = t^2$$

Exemplar Exam Question Answer

Substitute equation for bird into equation for coyote

Equations are for same point in time, so can substitute our equation for s_b into the equation for the coyote

$$s_b = t^2$$

$$80 + s_b = 10t + 1.25t^2$$

$$80 + t^2 = 10t + 1.25t^2$$

[1 Mark]

Solve quadratic equation

Bring all terms to right hand side

$$0 = 0.25t^2 + 10t - 80$$

Exemplar Exam Question Answer

Use quadratic formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-10 \pm \sqrt{10^2 - (4 \times 0.25 \times (-80))}}{2 \times 0.25} = 6.83 \ldots \text{ or } -46.83 \ldots$$

Time can't be a negative value

So $t = 6.83$ s

[1 Mark]

Projectiles

The **acceleration due to gravity** is always $g = 9.8(1)\mathrm{ms}^{-2}$ **vertically downwards**.

- A **projectile** can **experience** this **acceleration** while also **moving** with some **horizontal velocity**.

- **Vertical component** of **velocity equals 0** at **top** of **arc**.

- **Horizontal component** of **velocity** is **constant**.

Projectiles

To solve, **resolve** the **motion** into **vertical** and **horizontal components**.

- The **vertical component** undergoes **uniform acceleration** with $a = g$ so we use the **SUVAT equations**.

  Vertically:

  $a_V =$ $u_V =$ $s_V =$ $t_V =$ $v_V =$

- The **horizontal component** undergoes **uniform motion** (no **acceleration**) so we can use $\text{Distance} = \text{Speed} \times \text{Time}$.

  Horizontally:

  $a_H =$ $u_H =$ $s_H =$ $t_H =$

Lots of numbers for **calculation**, need to figure out what needs to be used when

Exemplar Exam Question

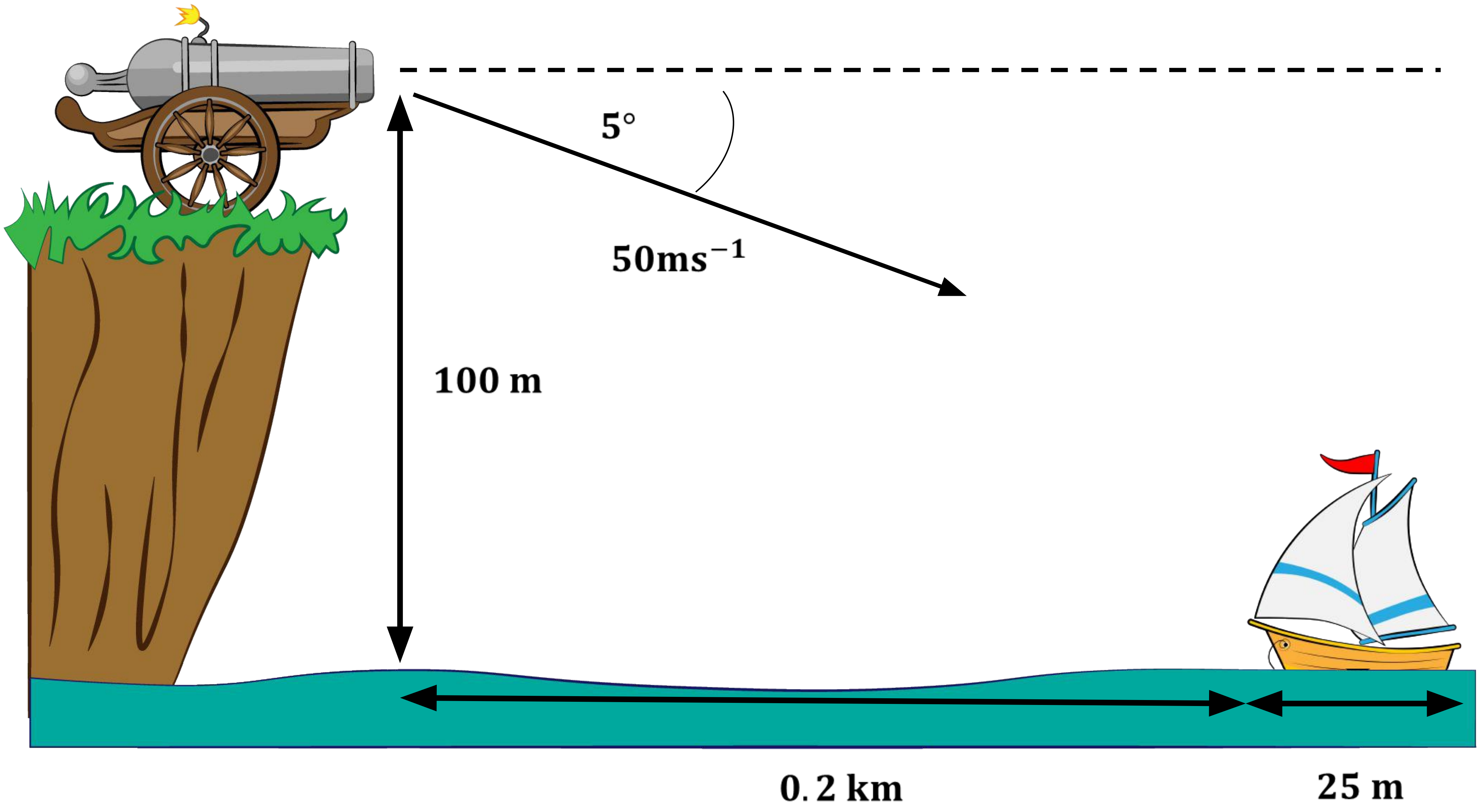
1) An island is being attacked by pirates. The soldiers on the island try to shoot the pirates by firing a cannon from a cliff, $\mathbf{100\ m}$ above sea level, at the edge of the island.
The ship and the cannon are both facing each other, with the front of the ship being $\mathbf{0.2\ km}$ away from the base of the cliff. The ship is $\mathbf{25}$m long.
The soldiers point the canon $\mathbf{5°}$ below the horizontal and fire a cannonball at $\mathbf{50\ ms^{-1}}$ as shown in the following diagram.
Neglecting air resistance and treating the height of the ship as negligible, calculate whether the ship will be hit by the cannonball

This means the **horizontal component** has **uniform motion**, **vertical** is only affected by **gravity** and you only need to **calculate** where the ball hits the sea

Determine where cannonball hits water and **compare** to position of ship

[5 marks]

5 step question


5°
50ms⁻¹
100 m
0.2 km
25 m

Exemplar Exam Question Answer

Need to resolve question into horizontal and vertical components.

First consider motion in vertical direction.

$$u_v = (50 \times \sin 5°)\ \text{ms}^{-1}$$

$$s_v = 100\ \text{m}$$

$$a_v = 9.8(1)\ \text{ms}^{-2}$$

$$t_v = ?$$

$$t_v = t_h$$

Exemplar Exam Question Answer

Select SUVAT equation to find t using variables given.

$u_v = (50 \times \sin 5°)\ \mathrm{ms^{-1}}$ $s_v = 100\ \mathrm{m}$ $a_v = 9.8(1)\ \mathrm{ms^{-2}}$ $t_v = ?$

$$s = ut + \frac{1}{2}at^2$$

$$100 = (50 \times \sin 5°)t + \frac{1}{2} \times 9.81 \times t^2$$

$$4.9(05)t^2 + 4.357\ldots t - 100 = 0$$

[1 Mark]

Exemplar Exam Question Answer

Form quadratic in t and solve for positive root of t.

$$4.9(05)t^2 + 4.357\ldots t - 100 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-4.357 \pm \sqrt{4.357^2 - 4 \times 4.9(05) \times (-100)}}{2 \times 4.9(05)}$$

$$= 4.09\ldots \mathrm{s} \text{ or } -4.98\ldots \mathrm{s}$$

[1 Mark]

Exemplar Exam Question Answer

$t_v = 4.09 \ldots = t_h$ **[1 Mark]**

Write down variables that are known in horizontal direction.

$t_h = 4.09 \ldots \text{s}$

$u_h = (50 \times \cos 5°)\ \text{ms}^{-1}$

$a_h = 0$

$s_h = ?$

Exemplar Exam Question Answer

As $a = 0$ do not need to use SUVAT.

$$\text{Displacement} = \text{Velocity} \times \text{Time}$$

$$s_h = u_h \times t_h$$

$$s_h = (50 \times \cos 5°)\ \text{ms}^{-1} \times 4.09\ldots\ \text{s}$$

$s_h = 204$ m to 3 s.f

[1 Mark]

Exemplar Exam Question Answer

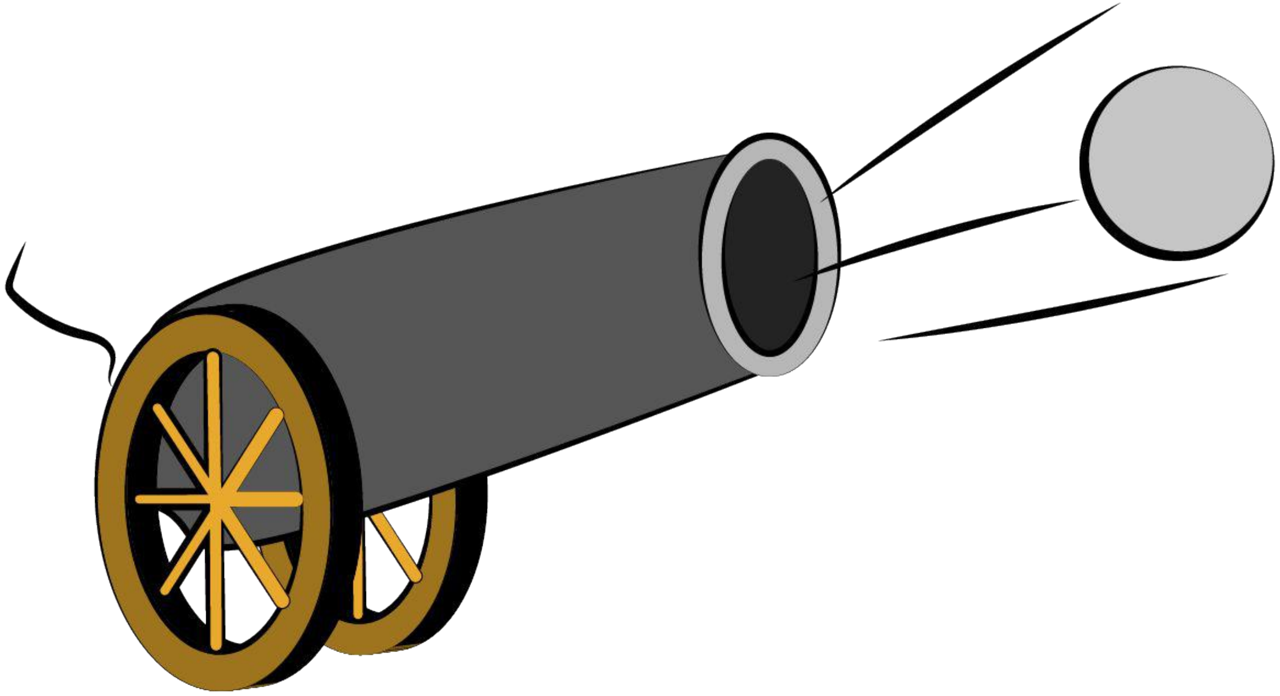
Determine whether cannonball hits pirate ship.

Canonball hits ship for $\mathbf{200} \leq s_h \leq (\mathbf{200} + \mathbf{25})$

$s_h = \mathbf{204\ m}$ therefore cannonball does hit pirate ship.

[1 Mark]

Forces



Specification Points - AQA

	Content
Q4	Use calculus in kinematics for motion in a straight line: $v = \frac{dr}{dt}, a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$, $r = \int v \, dt$, $v = \int a \, dt$; extend to 2 dimensions using vectors.

	Content
R1	Understand the concept of a force; understand and use Newton's first law.

	Content
R2	Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2D vectors); extend to situations where forces need to be resolved (restricted to 2 dimensions).

	Content
R3	Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g, and its value in SI units to varying degrees of accuracy. (The inverse square law for gravitation is not required and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location).

	Content
R4	Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2D vectors); application to problems involving smooth pulleys and connected particles; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.

	Content
R5	Understand and use addition of forces; resultant forces; dynamics for motion in a plane.

	Content
R6	Understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.

Specification Points – OCR A

3.02f 3.02g	Non uniform acceleration	f) Be able to use differentiation and integration with respect to time in one dimension to solve simple problems concerning the displacement, velocity and acceleration of a particle: $v = \frac{ds}{dt}$ $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ $s = \int v\,dt$ and $v = \int a\,dt$	g) Be able to extend the application of differentiation and integration to two dimensions using vectors: $\mathbf{x} = f(t)\mathbf{i} + g(t)\mathbf{j}$ $\mathbf{v} = \frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = f'(t)\mathbf{i} + g'(t)\mathbf{j}$ $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \frac{d^2\mathbf{x}}{dt^2} = f''(t)\mathbf{i} + g''(t)\mathbf{j}$ $\mathbf{x} = \int \mathbf{v}\,dt$ and $\mathbf{v} = \int \mathbf{a}\,dt$ *Questions set may involve either column vector or* **i, j** *notation.*

3.03f	Weight	f) Understand and be able to use the weight ($W = mg$) of a body to model the motion in a straight line under gravity. *e.g. A ball falling through the air.*
3.03g		g) Understand the gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy. *The value of g may be assumed to take a constant value of 9.8 ms^{-2} but learners should be aware that g is not a universal constant but depends on location in the universe.* *[The inverse square law for gravitation is not required.]*

3.03a	Newton's first law	a) Understand the concept and vector nature of a force. *A force has both a magnitude and direction and can cause an object with a given mass to change its velocity.* *Includes using directed line segments to represent forces (acting in at most two dimensions).* *Learners should be able to identify the forces acting on a system and represent them in a force diagram.*
3.03b		b) Understand and be able to use Newton's first law. *A particle that is at rest (or moving with constant velocity) will remain at rest (or moving with constant velocity) until acted upon by an external force.* *Learners should be able to complete a diagram with the force(s) required for a given body to remain in equilibrium.*

3.03c 3.03e	Newton's second law	c) Understand and be able to use Newton's second law ($F = ma$) for motion in a straight line for bodies of constant mass moving under the action of constant forces. *e.g. A car moving along a road, a passenger riding in a lift or a crane lifting a weight.* *For stage 1 learners, examples can be restricted to problems in which the forces acting on the body will be collinear, in two perpendicular directions or given as 2-D vectors.*	e) Be able to extend use of Newton's second law to situations where forces need to be resolved (restricted to two dimensions). *e.g. A force acting downwards on a body at a given angle to the horizontal or the motion of a body projected down a line of greatest slope of an inclined plane.*
3.03d		d) Understand and be able to use Newton's second law ($F = ma$) in simple cases of forces given as two dimensional vectors. *e.g. Find in vector form the force acting on a body of mass 2 kg when it is accelerating at $(4\mathbf{i} - 3\mathbf{j})$ m s^{-2}.* *Questions set involving vectors may involve either column vector notation* $\mathbf{F} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$ *or* **i, j** *notation* $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j}$.	

Specification Points – OCR A

3.03n 3.03o	Newton's third law (continued)	n) Be able to solve problems involving simple cases of equilibrium of forces on a particle in two dimensions using vectors, including connected particles and smooth pulleys. *e.g. Finding the required force* $\mathbf{F}$ *for a particle to remain in equilibrium when under the action of forces* $\mathbf{F}_1, \mathbf{F}_2, \ldots$ *For stage 1 learners, examples can be restricted to problems in which the forces acting on the body will be collinear, in two perpendicular directions or given as 2-D vectors.*	o) Be able to resolve forces for more advanced problems involving connected particles and smooth pulleys. *e.g. The motion of two particles, connected by a light inextensible string passing over a light pulley placed at the top of an inclined plane.*

3.03h 3.03l	Newton's third law	h) Understand and be able to use Newton's third law. *Every action has an equal and opposite reaction.* *Learners should understand and be able to use the concept that a system in which none of its components have any relative motion may be modelled as a single particle.*	l) Be able to extend use of Newton's third law to situations where forces need to be resolved (restricted to two dimensions).
3.03i		i) Understand and be able to use the concept of a normal reaction force. *Learners should understand and use the result that when an object is resting on a horizontal surface the normal reaction force is equal and opposite to the weight of the object. This includes knowing that when* $R = 0$ *contact is lost.*	
3.03j		j) Be able to use the model of a 'smooth' contact and understand the limitations of the model.	
3.03k 3.03m		k) Be able to use the concept of equilibrium together with one dimensional motion in a straight line to solve problems that involve connected particles and smooth pulleys. *e.g. A train engine pulling a train carriage(s) along a straight horizontal track or the vertical motion of two particles, connected by a light inextensible string passing over a fixed smooth peg or light pulley.*	m) Be able to use the principle that a particle is in equilibrium if and only if the sum of the resolved parts in a given direction is zero. *Problems may involve the resolving of forces, including cases where it is sensible to:* *1. resolve horizontally and vertically,* *2. resolve parallel and perpendicular to an inclined plane,* *3. resolve in directions to be chosen by the learner, or* *4. use a polygon of forces.*

Specification Points – OCR MEI

MF1	Understand the language relating to forces.
F2	Know that the acceleration due to gravity is not a universal constant but depends on location in the universe. Know that on earth, the acceleration due to gravity is often modelled to be a constant, g m s^{-2}.
F3	Be able to identify the forces acting on a system and represent them in a force diagram. Understand the difference between external and internal forces and be able to identify the forces acting on part of the system.
F4	Be able to find the resultant of several concurrent forces when the forces are parallel or in two perpendicular directions or in simple cases of forces given as 2-D vectors in component form.
F5	Understand the concept of equilibrium and know that a particle is in equilibrium if and only if the vector sum of the forces acting on it is zero in the cases where the forces are parallel or in two perpendicular directions or in simple cases of forces given as 2-D vectors in component form.

MF6	Be able to resolve a force into components and be able to select suitable directions for resolution. Be able to find the resultant of several concurrent forces by resolving and adding components.
F7	Know that a particle is in equilibrium if and only if the resultant of the forces acting on it is zero. Know that a body is in equilibrium under a set of concurrent forces if and only if their resultant is zero.
F8	Know that vectors representing a set of forces in equilibrium sum to zero. Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium.
F9	Be able to formulate and solve equations for a particle in equilibrium: by resolving forces in suitable directions; by drawing and using a polygon of forces.
F10	Understand that the overall contact force between surfaces may be expressed in terms of a frictional force and a normal contact force and be able to draw an appropriate force diagram. Understand that the normal contact force cannot be negative.
F11	Understand that the frictional force may be modelled by $F \leq \mu R$ and that friction acts in the direction to oppose sliding. Model friction using $F = \mu R$ when sliding occurs.
F12	Be able to apply Newton's Laws to problems involving friction.

Mn1	Know and understand the meaning of Newton's three laws.
n2	Understand the term equation of motion.
n3	Be able to formulate the equation of motion for a particle moving in a straight line when the forces acting are parallel or in two perpendicular directions or in simple cases of forces given as 2-D vectors in component form.
n4	Be able to model a system as a set of connected particles.
n5	Be able to formulate the equations of motion for the individual particles within the system.
n6	Know that a system in which none of its components have any relative motion may be modelled as a single particle with the mass of the system.

Specification Points - Edexcel

7.4	**Use calculus in kinematics for motion in a straight line:** $v = \frac{dr}{dt}$, $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$ $r = \int v \ dt$, $v = \int a \ dt$ Extend to 2 dimensions using vectors.	**The level of calculus required will be consistent with that in Sections 7 and 8 in Paper 1** and Sections 6 and 7 in Paper 2. Differentiation and integration of a vector with respect to time. e.g. Given $\mathbf{r} = t^2\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}$, find $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ at a given time.
8.4	**Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles;** resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.	**Connected particle problems could include problems with particles in contact e.g. lift problems.** Problems may be set where forces need to be resolved, e.g. at least one of the particles is moving on an inclined plane.
8.5	Understand and use addition of forces; resultant forces; dynamics for motion in a plane.	Students may be required to resolve a vector into two components or use a vector diagram, e.g. problems involving two or more forces, given in magnitude-direction form.

8.1	**Understand the concept of a force; understand and use Newton's first law.**	**Normal reaction, tension, thrust or compression, resistance.**
8.2	**Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors);** extend to situations where forces need to be resolved (restricted to 2 dimensions).	**Problems will involve motion in a straight line with constant acceleration in scalar form, where the forces act either parallel or perpendicular to the motion.** Extend to problems where forces need to be resolved, e.g. a particle moving on an inclined plane. **Problems may involve motion in a straight line with constant acceleration in vector form, where the forces are given in i – j form or as column vectors.**
8.3	**Understand and use weight and motion in a straight line under gravity; gravitational acceleration, *g*, and its value in S.I. units to varying degrees of accuracy.**	**The default value of g will be 9.8 m s^{-2} but some questions may specify another value, e.g. g = 10 m s^{-2}** **The inverse square law for gravitation is not required and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location.**

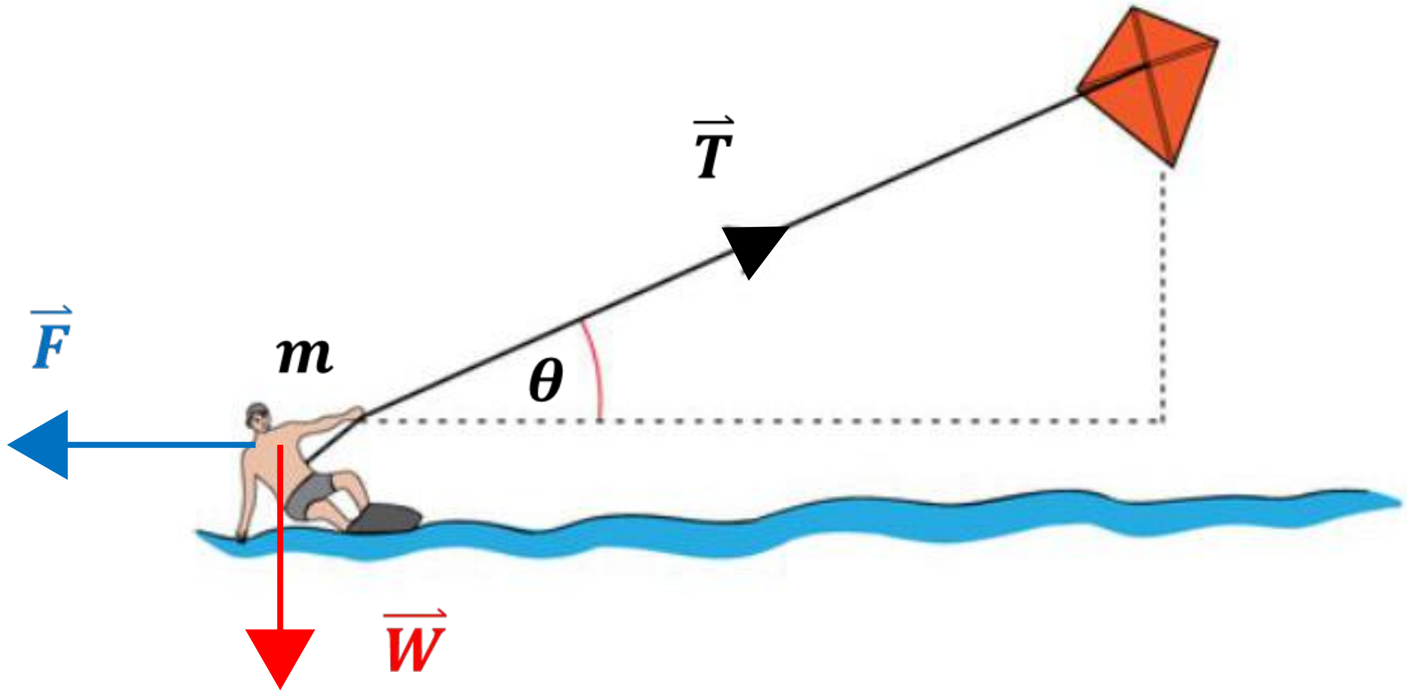
8.6	Understand and use the $F \le \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.	An understanding of $F = \mu R$ when a particle is moving. An understanding of $F \le \mu R$ in a situation of equilibrium.

Dynamics and Equilibrium

- To **determine** the **acceleration** of an **object** we **resolve** the **forces acting** on it.
- We can take **horizontal** and **vertical** **components** to find the **components** of $\vec{a}$.

Right:

Up:

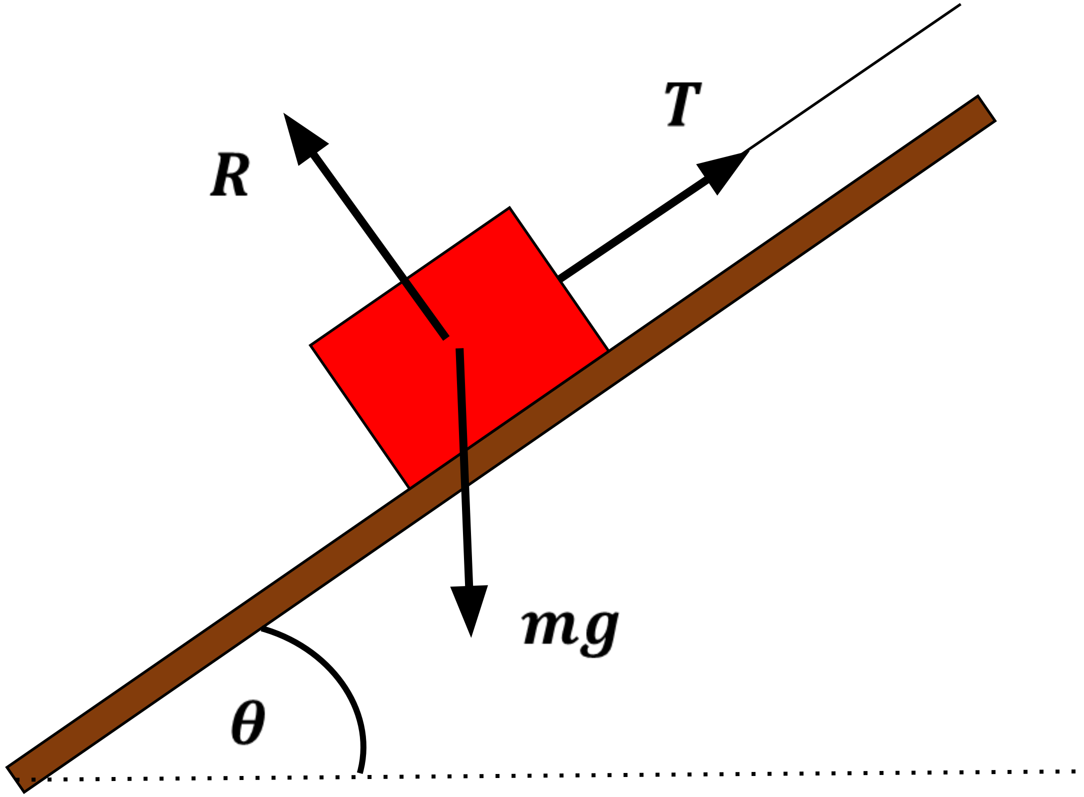


Dynamics and Equilibrium

For an object **at rest** on a **slope** we can **resolve forces parallel** to and **perpendicular** to the **slope**.

- **Parallel** to **slope**:

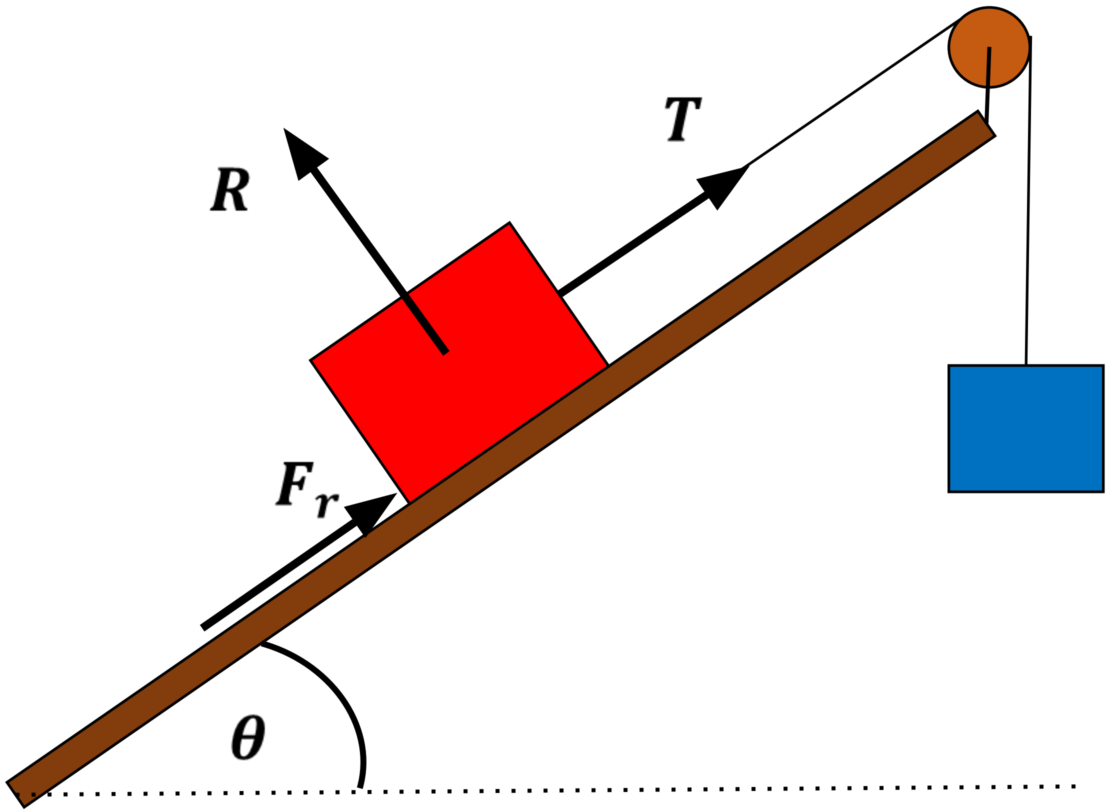
- **Perpendicular** to **slope**:



Dynamics and Equilibrium

Friction (F_r) can **act up** or **down** the **slope** to **resist motion**.

The **tension** in a **light inextensible string** is the **same** at **both ends**



Need to **resolve forces**

Exemplar Exam Question

Lots of **information**. How best to **break this down**?

•

What **forces** will be present?

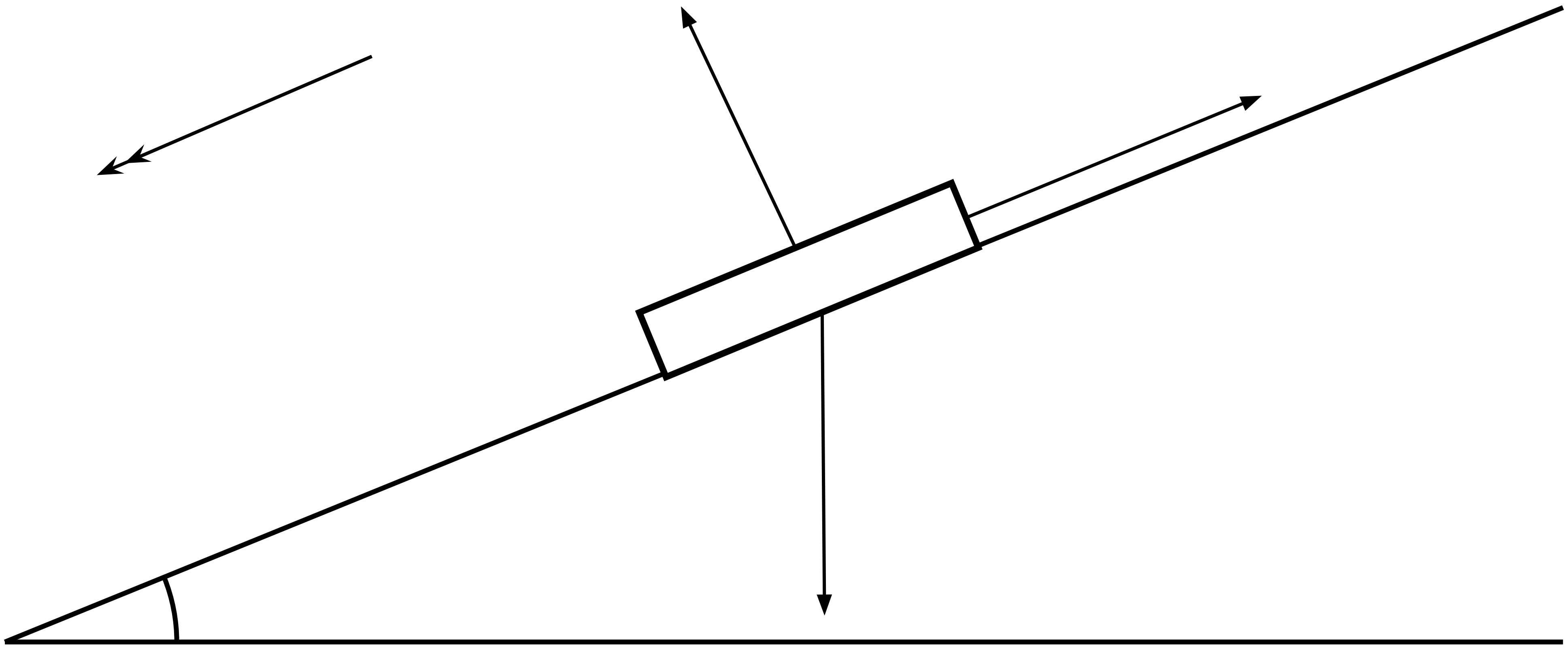
What do these **modelling assumptions** mean?

4 main steps to calculation

Do we need to **calculate** this value?

Exemplar Exam Question Answer

Draw a diagram of the given context



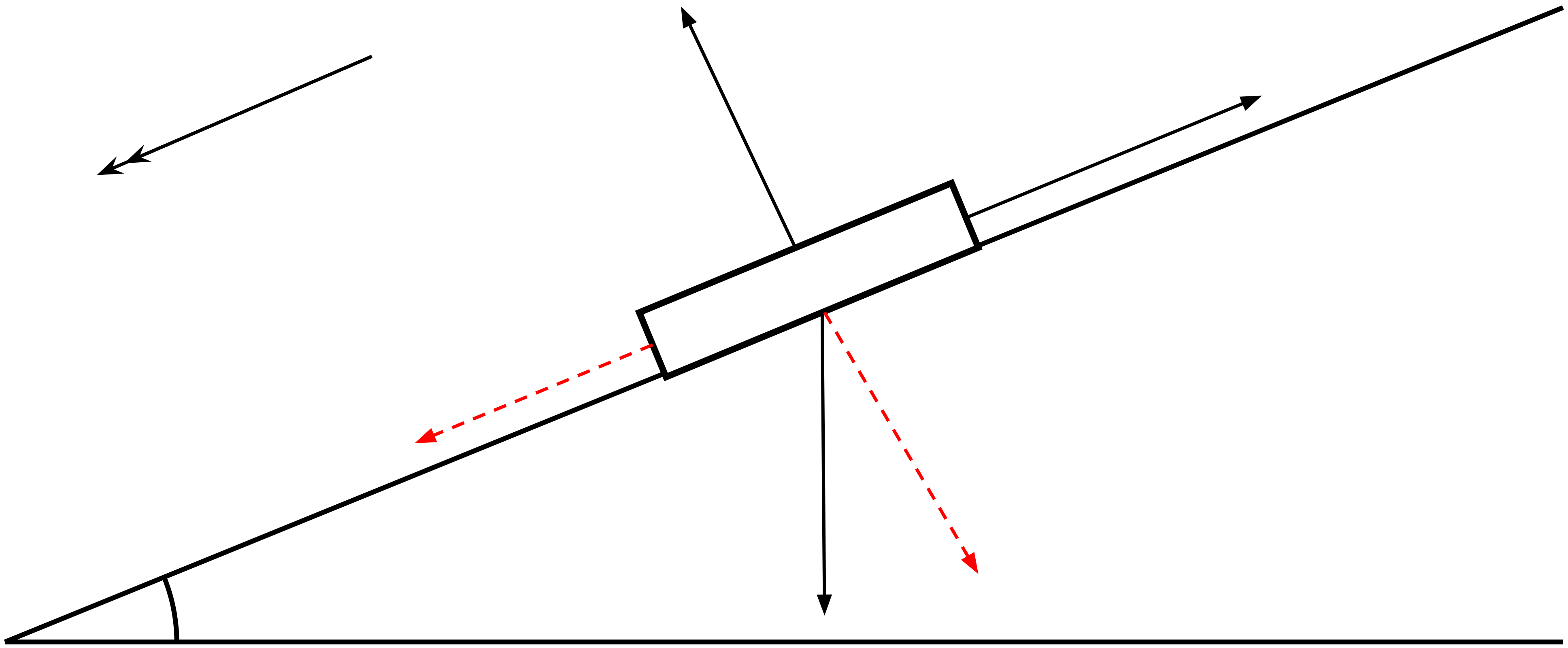
Exemplar Exam Question Answer

Resolve weight into components perpendicular and parallel to the slope.

[1 Mark]

Exemplar Exam Question Answer

Update diagram with components of weight



Exemplar Exam Question Answer

Resolve forces perpendicular to the plane.

Calculate magnitude of frictional force

[1 Mark]

Exemplar Exam Question Answer

Resolve forces parallel to the plane to find resultant force

[1 Mark]

Exemplar Exam Question Answer

Use Newton's Second Law to calculate acceleration of tile

[1 Mark]

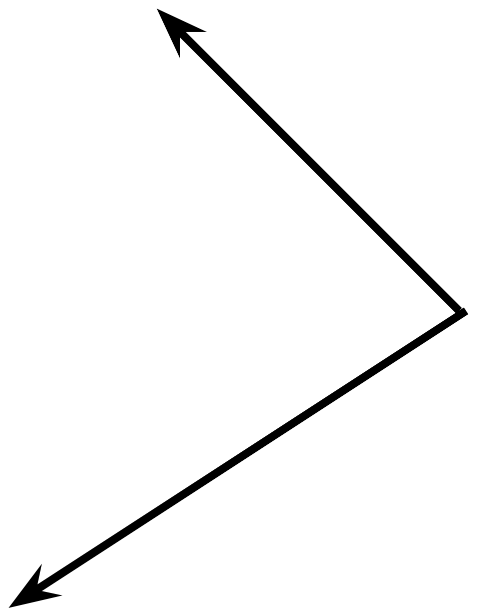
MINI MOCK PAPER

- # Exam Question

Exam Question Answer

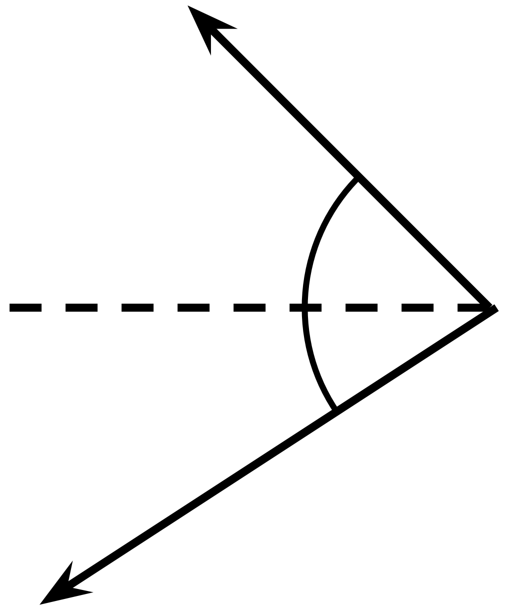
Calculate the two vectors in column format

[1 Mark]



Exam Question Answer

Calculate the angle each vector makes with the horizontal



[1 Mark]

Add the angles together

[1 Mark]

Exam Question

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Exam Question Answer

Use Newton’s Second Law to find the acceleration due to the thrust force.

[1 Mark]

Exam Question Answer

List the given information to determine which SUVAT equation should be used

Exam Question Answer

Substitute values and rearrange equation

[1 Mark]

Exam Question Answer

Solve using the quadratic formula

[2 Marks]