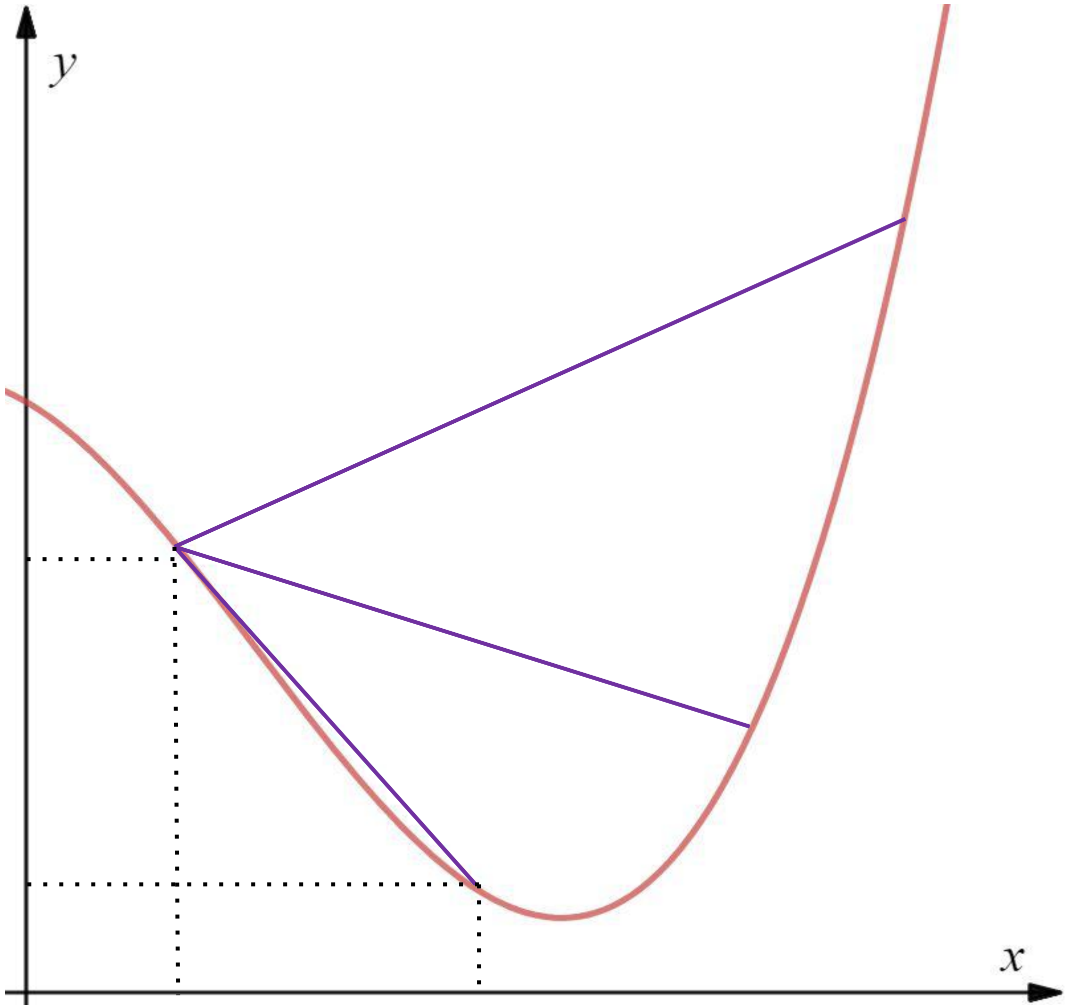
Pure Mathematics: Differentiation



Material Covered

Differentiating Polynomials and Related Functions

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Specification Points - AQA

	Content
G1	Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives; differentiation from first principles for small positive integer powers of x and for $\sin x$ and $\cos x$ Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.

	Content
G4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.

	Content
G6	Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand).

Specification Points – OCR A

a) Understand and be able to use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y).

b) Understand and be able to use the gradient of the tangent at a point where $x = a$ as:
1. the limit of the gradient of a chord as x tends to a
2. a rate of change of y with respect to x.

t) Be able to construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population growth and modelling the relationship between price and demand).

r) Be able to differentiate using the chain rule, including problems involving connected rates of change and inverse functions.

In particular, learners should be able to use the following relations:

$\frac{dy}{dx} = 1 \div \frac{dx}{dy}$ *and* $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

g) Be able to show differentiation from first principles for small positive integer powers of x.

In particular, learners should be able to use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ *including the notation.*

[*Integer powers greater than 4 are excluded.*]

h) Be able to show differentiation from first principles for $\sin x$ and $\cos x$.

Specification Points – OCR MEI

Mc1	Know and use that the gradient of a curve at a point is given by the gradient of the tangent at the point.		
c2	Know and use that the gradient of the tangent at a point A on a curve is given by the limit of the gradient of chord AP as P approaches A along the curve.		
c3	Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y). Know that the gradient function $\frac{dy}{dx}$ gives the gradient of the curve and measures the rate of change of y with respect to x.	Be able to deduce the units of rate of change for graphs modelling real situations. The term derivative of a function.	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ $f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$

c14	Be able to differentiate composite functions using the chain rule.	$y = f(u), \quad u = g(x), \quad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ *or* $\{f[g(x)]\}' = f'[g(x)]g'(x)$
c15	Be able to find rates of change using the chain rule, including connected rates of change and differentiation of inverse functions.	$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$

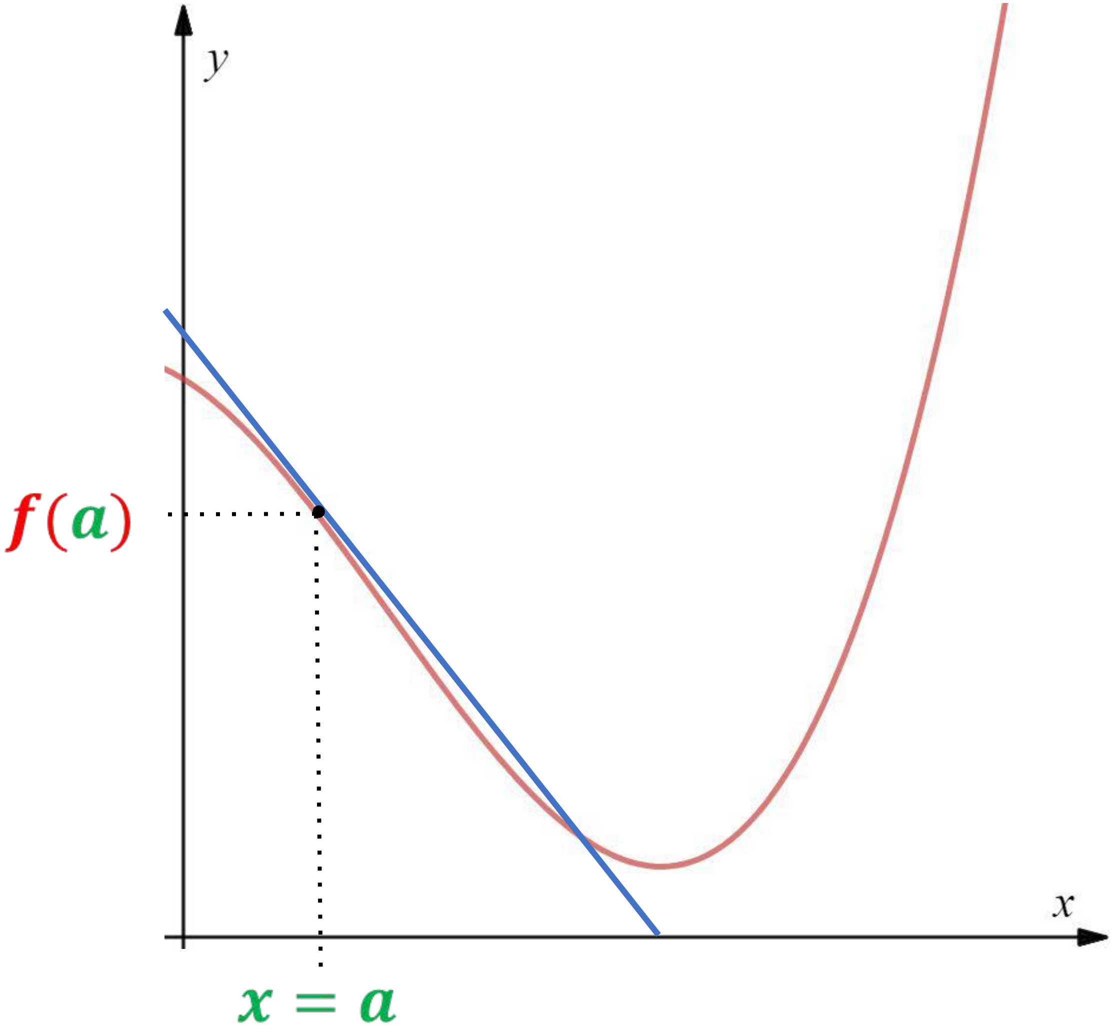
Specification Points - Edexcel

7.1	**Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change**	**Know that $\frac{dy}{dx}$ is the rate of change of y with respect to x.** **The notation $f'(x)$ may be used for the first derivative and $f''(x)$ may be used for the second derivative.**
	sketching the gradient function for a given curve	**Given for example the graph of $y = f(x)$, sketch the graph of $y = f'(x)$ using given axes and scale. This could relate speed and acceleration for example.**
	second derivatives	
	differentiation from first principles for small positive integer powers of x and for $\sin x$ and $\cos x$	**For example, students should be able to use, for $n = 2$ and $n = 3$, the gradient expression** $\lim_{h \to 0}\left(\frac{(x+h)^n - x^n}{h}\right)$ **Students may use δx or h**

7.4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.	Differentiation of cosec x, cot x and sec x. Differentiation of functions of the form $x = \sin y, x = 3\tan 2y$ and the use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$ Use of connected rates of change in models, e.g. $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos^2 x$ and $\tan^2 2x$.
7.6	Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand).	Set up a differential equation using given information which may include direct proportion.

Derivative of a Function

The **derivative** $\left(\frac{dy}{dx}\right)$ of a **function** $f(x)$ at a specific **value** of $x = a$ is **equal** to the **gradient** of the **tangent** to $f(x)$ at $x = a$.



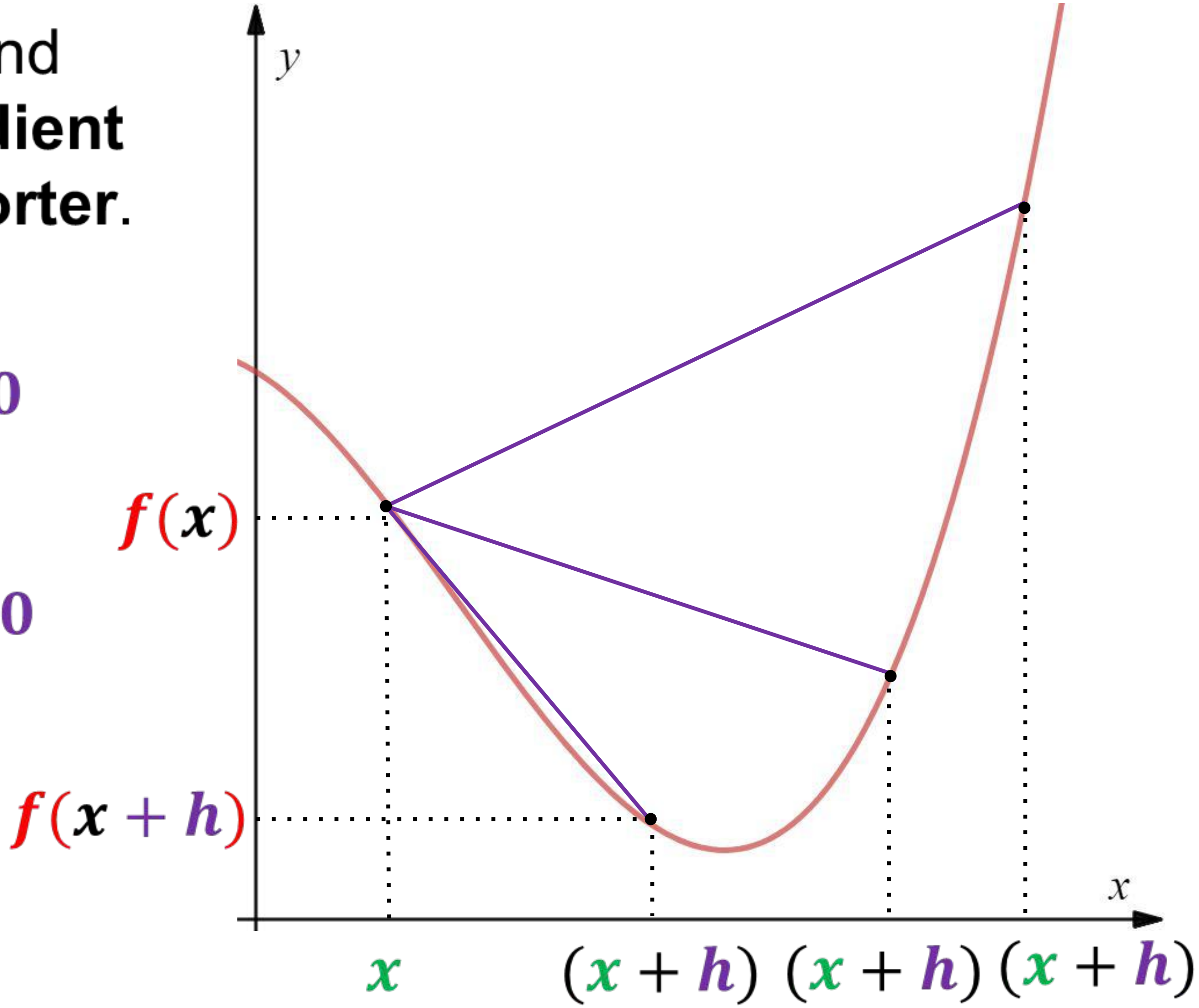
Differentiation from First Principles

The **gradient** of a **chord** joining **2 points** x and $(x + h)$ on a **curve** $f(x)$ approaches the **gradient** (m) of the **tangent** to the **curve** as it gets **shorter**.

$$m = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{(x+h) - x} \text{ as } h \to 0$$

$$= \frac{f(x+h) - f(x)}{h} \text{ as } h \to 0$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



Derivative of a Polynomial

The **derivative** $f'(x)$ of a **polynomial function** $f(x)$ is given by:

$$f(x) = x^n \quad \Rightarrow \quad f'(x) = nx^{n-1}$$

The **derivative** $f'(x)$ of a **constant** c is given by:

$$f(x) = c \quad \Rightarrow \quad f'(x) = 0$$

Exemplar Exam Question

1) For $f(x) = 4x^2 - 1$ derive $f'(x)$ using differentiation from first principles.

[4 marks]

Will have to **show every step** of **working**.

Need to use **first principles formula**.

4 marks, 4 key steps

Exemplar Exam Question Answer

Recall equation for First Principles:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = 4x^2 - 1 \qquad f(x+h) = 4(x+h)^2 - 1$$

$$= 4(x^2 + 2xh + h^2) - 1$$

$$= 4x^2 + 8xh + 4h^2 - 1$$

[1 mark]

Exemplar Exam Question Answer

Substitute $f(x+h)$ and $f(x)$ into equation:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(4x^2 + 8xh + 4h^2 - 1) - (4x^2 - 1)}{h}$$

[1 mark]

$$= \lim_{h \to 0} \frac{8xh + 4h^2}{h} \qquad = \lim_{h \to 0} 8x + 4h$$

[1 mark]

Exemplar Exam Question Answer

Resolve limit.

$$f'(x) = \lim_{h \to 0}(8x + 4h)$$

$$= 8x + 0$$

$$= 8x$$

[1 mark]

Chain Rule

The **chain rule** is used to **differentiate composite functions** – a **composite function** is a **function** of another **function**.

$$y = f(g(x))$$

$$\frac{dy}{dx} = g'(x) \times f'(g(x))$$

Chain Rule

To answer **questions** involving the **chain rule**, first **identify** the **2 functions** $f(u)$ and $g(x)$ and their **derivatives** $f'(u)$ and $g'(x)$.

$$y = \sqrt{x^4 + 1}$$

$f(u) =$

$g(x) =$

$f'(u) =$

$g'(x) =$

Exemplar Exam Question

Recall **formula** for **chain rule**.

1) Find $\frac{dy}{dx}$ when:

$$y = (3x^2 - 1)^{10}$$

[2 marks]

Identify the two functions here

2 marks, 2 key steps

Exemplar Exam Question Answer

Recall chain rule: $$\frac{d}{dx}f(g(x)) = g'(x) \times f'(g(x))$$

Identify $f(u)$ and $g(x)$:

$f(u) = u^{10}$ $\quad$ $g(x) = 3x^2 - 1$

Exemplar Exam Question Answer

Identify $f'(u)$ and $g'(x)$:

$f'(u) = 10u^9 \qquad g'(x) = 6x$

[1 mark]

Substitute into chain rule expression: $\frac{d}{dx}f(g(x)) = g'(x) \times f'(g(x))$

$$\frac{dy}{dx} = 6x \times 10(3x^2 - 1)^9 = 60x(3x^2 - 1)^9$$

[1 mark]

Rates of Change

It is **possible** to **express** the **chain rule** in the **following form**:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

- We know that $\frac{dy}{dx}$ gives the **rate of change** of y **with respect to** x.
- We can use the **chain rule** to find the **rate of change** of other **variables**.

Rates of Change

Consider the **rate of change** of **volume** (V) of a **sphere with respect to time** (t) if the **rate of change** of its **radius** (r) is a **constant** k.

Inverse Chain Rule

We can use the following **identity** to **differentiate inverse functions**

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

Exemplar Exam Question

Will need to use **equation** for **volume** of **conical funnel** given in **question**.

1) Water flows out of a conical funnel at a rate proportional to the square root of the volume V of water left in the tank. Determine an equation to model the rate of change of the depth of the tank h in terms of h and any other constant terms. The volume of water in the conical funnel is given by $V = \frac{\pi h^3}{12}$.

[4 marks]

Question involving **rates of change**. We may need to **apply** the **chain rule**.

Expect **answer** to contain these **variables**.

Exemplar Exam Question Answer

Use the chain rule to write an equation for the rate of change of depth with respect to time.

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

Need to determine $\frac{dh}{dV}$ and $\frac{dV}{dt}$.

Exemplar Exam Question Answer

Determine equation for $\frac{dV}{dt}$ from the question.

$$\frac{dV}{dt} \propto \sqrt{V} \qquad \Rightarrow \frac{dV}{dt} = -k\sqrt{V}$$

To determine $\frac{dh}{dV}$ apply the equation for volume the conical funnel.

$$V = \frac{\pi h^3}{12}$$

$$\frac{dV}{dh} = \frac{\pi}{4}h^2$$

[1 mark]

Exemplar Exam Question Answer

Write equation for the rate of change of depth with respect to time.

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$\frac{dh}{dV} = \frac{1}{\frac{dV}{dh}} = \frac{4}{\pi h^2}$$ **[1 mark]**

$$\frac{dV}{dt} = -k\sqrt{V} \quad = -k\sqrt{\frac{\pi h^3}{12}}$$ **[1 mark]**

Exemplar Exam Question Answer

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi h^2} \times -k\sqrt{\frac{\pi h^3}{12}} = -4k\sqrt{\frac{\pi h^3}{12\pi^2 h^2}}$$

$$= -2k\sqrt{\frac{1}{3h\pi}}$$

[1 mark]

Differentiating Advanced Functions

$$\frac{dy}{dx} = \frac{f'(x) \times g(x) - f(x) \times g'(x)}{\left(g(x)\right)^2}$$

Specification Points - AQA

	Content
G2	Differentiate x^n , for rational values of n , and related constant multiples, sums and differences. Differentiate e^{kx} and a^{kx} , sin kx , cos kx , tan kx and related sums, differences and constant multiples. Understand and use the derivative of ln x

	Content
G4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.

Specification Points – OCR A

j)	Be able to differentiate e^{kx} and a^{kx}, and related sums, differences and constant multiples.
k)	Be able to differentiate $\sin kx$, $\cos kx$, $\tan kx$ and related sums, differences and constant multiples.
l)	Understand and be able to use the derivative of $\ln x$.
q)	Be able to differentiate using the product rule and the quotient rule.

Specification Points – OCR MEI

Mc10	Be able to differentiate e^{kx}, a^{kx} and $\ln x$.	Including related sums, differences and constant multiples.
c11	Be able to differentiate the trigonometrical functions: $\sin kx$; $\cos kx$; $\tan kx$ for x in radians.	Including their constant multiples, sums and differences. Differentiation from first principles for $\sin x$ and $\cos x$.
c12	Be able to differentiate the product of two functions.	The product rule: $y = uv$, $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ *Or* $[f(x)g(x)]' = f(x)g'(x) + f'(x)g(x)$
c13	Be able to differentiate the quotient of two functions.	$y = \frac{u}{v}, \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ *Or* $\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Specification Points - Edexcel

7.4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.	Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$. Differentiation of functions of the form $x = \sin y$, $x = 3\tan 2y$ and the use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$ Use of connected rates of change in models, e.g. $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos^2 x$ and $\tan^2 2x$.

7.2	**Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.** Differentiate e^{kx} and a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$ and related sums, differences and constant multiples. Understand and use the derivative of $\ln x$	**For example, the ability to differentiate expressions such as $(2x+5)(x-1)$ and $\frac{x^2+3x-5}{4x^{\frac{1}{2}}}$, $x > 0$, is expected.** Knowledge and use of the result $\frac{d}{dx}(a^{kx}) = ka^{kx} \ln a$ is expected.

Product Rule

The **product rule** is used to **differentiate** a function $f(x)$ **multiplied** by another **function** $g(x)$.

$$y = f(x) \times g(x)$$

$$\frac{dy}{dx} = f'(x) \times g(x) + f(x) \times g'(x)$$

We can also write the **product rule** in terms of the **functions** u and v:

$$(u \times v)' = u' \times v + u \times v'$$

Quotient Rule

The **quotient rule** is used to **differentiate** a function $f(x)$ **divided** by another **function** $g(x)$.

$$y = \frac{f(x)}{g(x)} \qquad \frac{dy}{dx} = \frac{f'(x) \times g(x) - f(x) \times g'(x)}{\left(g(x)\right)^2}$$

We can also write the **quotient rule** in terms of the **functions** u and v:

$$\left(\frac{u}{v}\right)' = \frac{u' \times v - u \times v'}{v^2}$$

Exemplar Exam Question

1) Use the product rule to differentiate the following function:

3 marks, 3 key steps

$$f(x) = x^3\sqrt{2x+1}$$

[3 marks]

Recall formula for **product rule**.

Identify u and v.

Exemplar Exam Question Answer

$$f(x) = x^3\sqrt{2x+1}$$

Recall product rule: $(u \times v)' = u' \times v + u \times v'$

Identify u and v: $u = x^3$ $\quad v = (2x+1)^{\frac{1}{2}}$

Identify u' and v': $u' = 3x^2$ $\quad v' = \frac{1}{2} \times 2 \times (2x+1)^{-\frac{1}{2}}$

$$= (2x+1)^{-\frac{1}{2}}$$

[2 marks]

Exemplar Exam Question Answer

Substitute into product rule expression: $(u \times v)' = u' \times v + u \times v'$

$$f'(x) = 3x^2 \times (2x+1)^{\frac{1}{2}} + x^3 \times (2x+1)^{-\frac{1}{2}}$$

$$= 3x^2\sqrt{2x+1} + \frac{x^3}{\sqrt{2x+1}}$$

[1 mark]

Differentiating Trigonometric Functions

Differentiating a **trigonometric function** gives another **trigonometric function**.

$y = \sin(x)$

$y = \cos(x)$

$y = \tan(x)$

Differentiating Trigonometric Functions

We can use the **chain rule** to **differentiate** more **complex trigonometric functions**.

$$y = f(g(x)) \qquad \frac{dy}{dx} = g'(x) \times f'(g(x))$$

$$y = \sin(kx)$$

$$y = \sec(x)$$

Differentiating Trigonometric Functions

The following **table** gives the **derivatives** of **trigonometric functions** that you will need to know:

Function	$\sin x$	$\cos x$	$\tan x$
Derivative	$\cos x$	$-\sin x$	$\sec^2 x$
Function	$\operatorname{cosec} x$	$\sec x$	$\cot x$
Derivative		$\tan x \sec x$	

Exemplar Exam Question

2 marks,
2 key steps

1) Use the quotient rule to differentiate the following equation with respect to x:

$$y = \frac{\tan x}{x}$$

[2 marks]

Recall formulae for **differentiating trigonometric functions**.

Identify two functions to input to **quotient rule formula**

Exemplar Exam Question Answer

$$y = \frac{\tan x}{x}$$

Recall quotient rule: $\left(\frac{u}{v}\right)' = \frac{u' \times v - u \times v'}{v^2}$

Identify u and v: $u = \tan x \qquad v = x$

Identify u' and v': $u' = \sec^2 x$

$v' = 1$

[1 mark]

Exemplar Exam Question Answer

Substitute into quotient rule expression: $\left(\frac{u}{v}\right)' = \frac{u' \times v - u \times v'}{v^2}$

$$\frac{dy}{dx} = \frac{(x\sec^2 x - \tan x)}{x^2}$$

[1 mark]

Differentiating Exponentials

The function $y = e^x$ is unique as **differentiating** it gives the **same result**.

We can use the **chain rule** to **differentiate** more **complex exponential functions**.

$$y = e^{kx}$$

$$y = a^x$$

Differentiating Logarithms

Differentiating a **function** containing the **natural logarithm** gives:

$$y = \ln(f(x))$$

We can use the **logarithm rules** to **differentiate** more **complex logarithmic functions**.

$$y = \ln(kx)$$

$$y = \ln(x^x)$$

Exemplar Exam Question

Will likely find an **inequality for x**

1) By finding $\frac{dy}{dx}$, give the values of x for which the function $y = 3^{x^2}$ is an increasing function.

[5 marks]

Recall formulae for **differentiating exponential functions**.

Recall properties of $\frac{dy}{dx}$ for an **increasing function**.

Exemplar Exam Question Answer

Determine $\frac{dy}{dx}$.

Recall chain rule: $$\frac{d}{dx}f(g(x)) = g'(x) \times f'(g(x))$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x^2}\right)$$

Exemplar Exam Question Answer

Identify $f(u)$ and $g(x)$:

$f(u) = 3^u$ $\quad$ $g(x) = x^2$

$f'(u) = \ln(3) \times 3^u$ $\quad$ $g'(x) = 2x$ **[1 mark]**

Substitute into chain rule expression: $\frac{d}{dx} f(g(x)) = g'(x) \times f'(g(x))$

$$\frac{d}{dx}\left(3^{x^2}\right) = 2x \times \ln(3) \times 3^{x^2}$$ **[1 mark]**

Exemplar Exam Question Answer

$$\frac{dy}{dx} = 2x \times \ln(3) \times 3^{x^2}$$ **[1 mark]**

Determine where y is increasing.

$y = 3^{x^2}$ is increasing where $\frac{dy}{dx} > 0$.

$$\frac{dy}{dx} > 0 \Rightarrow 2x \times \ln(3) \times 3^{x^2} > 0$$ **[1 mark]**

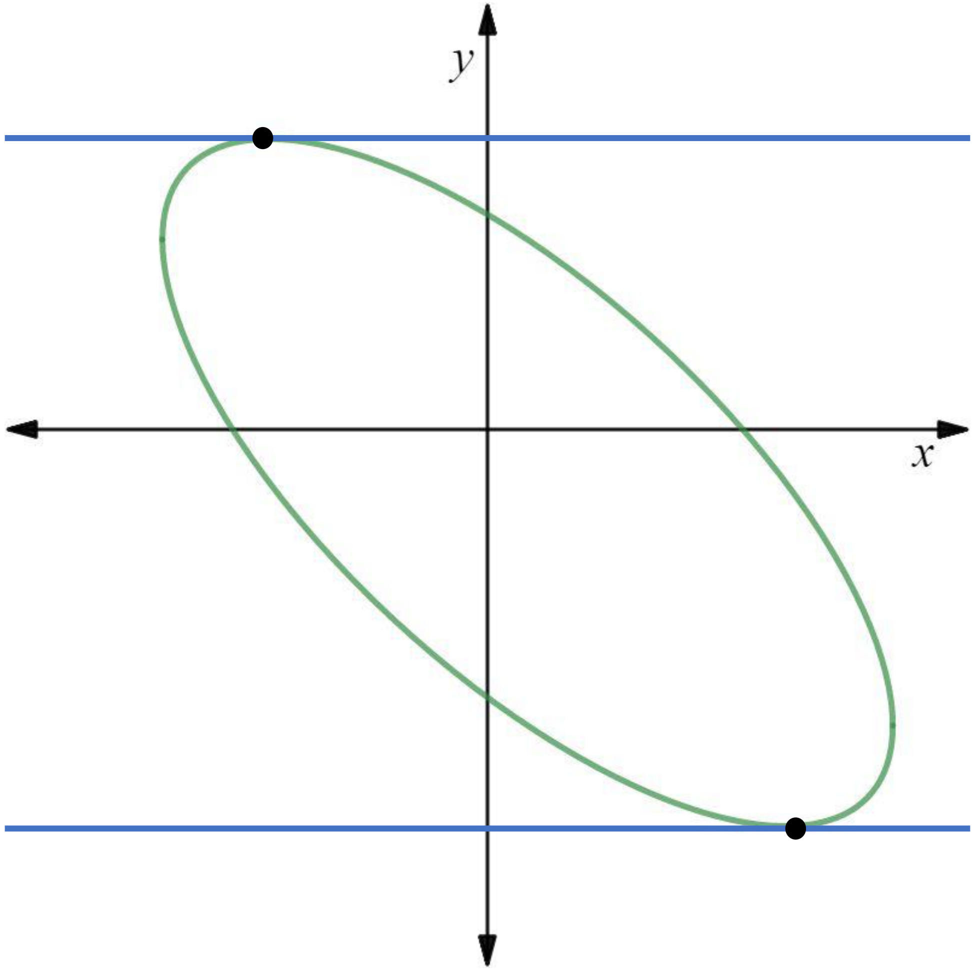
Exemplar Exam Question Answer

$3^{x^2} > 0$ for all values of x.

Therefore $\dfrac{dy}{dx} = 2x \times \ln(3) \times 3^{x^2} > 0$ for $x > 0$.

Therefore $y = 3^{x^2}$ is increasing for $x > 0$. **[1 mark]**

Implicit Differentiation



Specification Points - AQA

	Content
G5	Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only.

Specification Points – OCR A

s) Be able to differentiate simple functions and relations defined implicitly or parametrically for the first derivative only.

They should be able to find the gradient at a point on a curve and to use this to find the equations of tangents and normals, and to solve associated problems.

Includes differentiation of functions defined in terms of a parameter using the chain rule.

Specification Points – OCR MEI

c16	Be able to differentiate a function or relation defined implicitly.	e.g. $(x+y)^2 = 2x$.

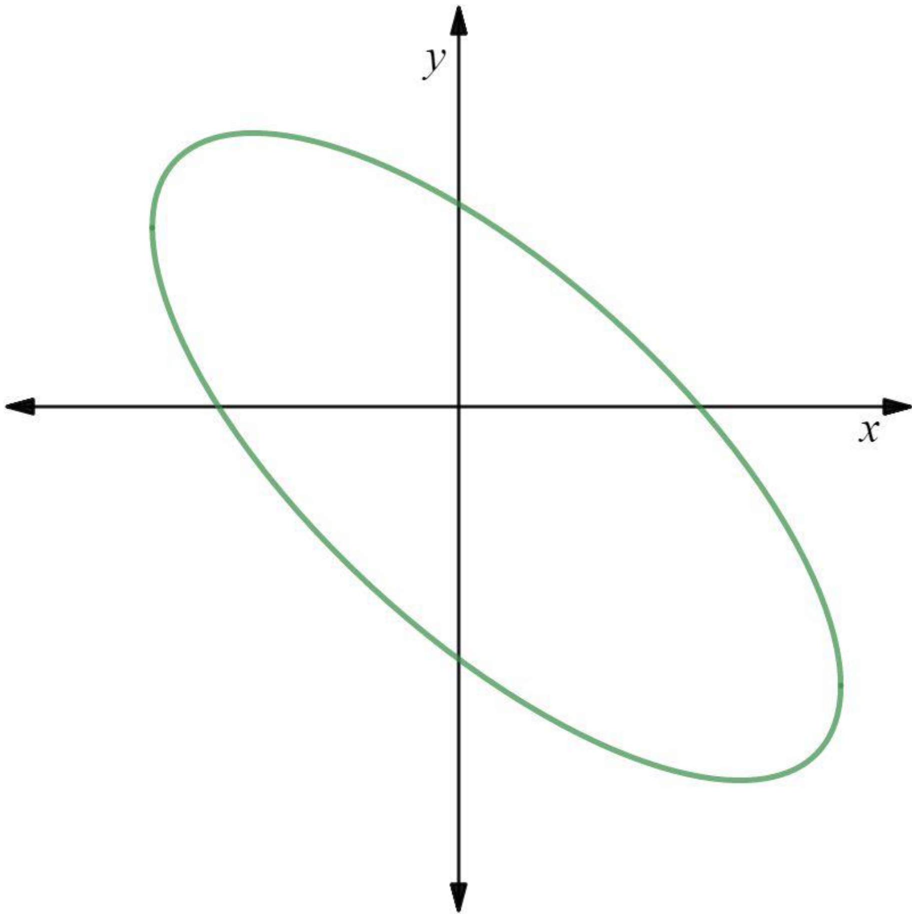
Specification Points - Edexcel

7.5	Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.

Implicit Differentiation

Some **curves** are given by **equations** containing **functions** of y, **functions** of x and **functions of both** x **and** y.

$$x^2 + 2xy + 2y^2 = 4$$



- To find the **maximum** and **minimum** y **values** of the **curve** we need to **find** where $\frac{dy}{dx} = 0$.

Implicit Differentiation

- To **find** $\frac{dy}{dx}$ we must **differentiate** the **equation** of the **curve implicitly**.
- To **differentiate** an **expression implicitly**, **differentiate** each **term individually**.

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(2xy) + \frac{d}{dx}(2y^2) = \frac{d}{dx}(4)$$

Implicit Differentiation

- To **differentiate** a **function** of y use the **chain rule**.

$$\frac{d}{dx}f(u) = u' \times f'(u)$$

$$\frac{d}{dx}(2y^2)$$

Implicit Differentiation

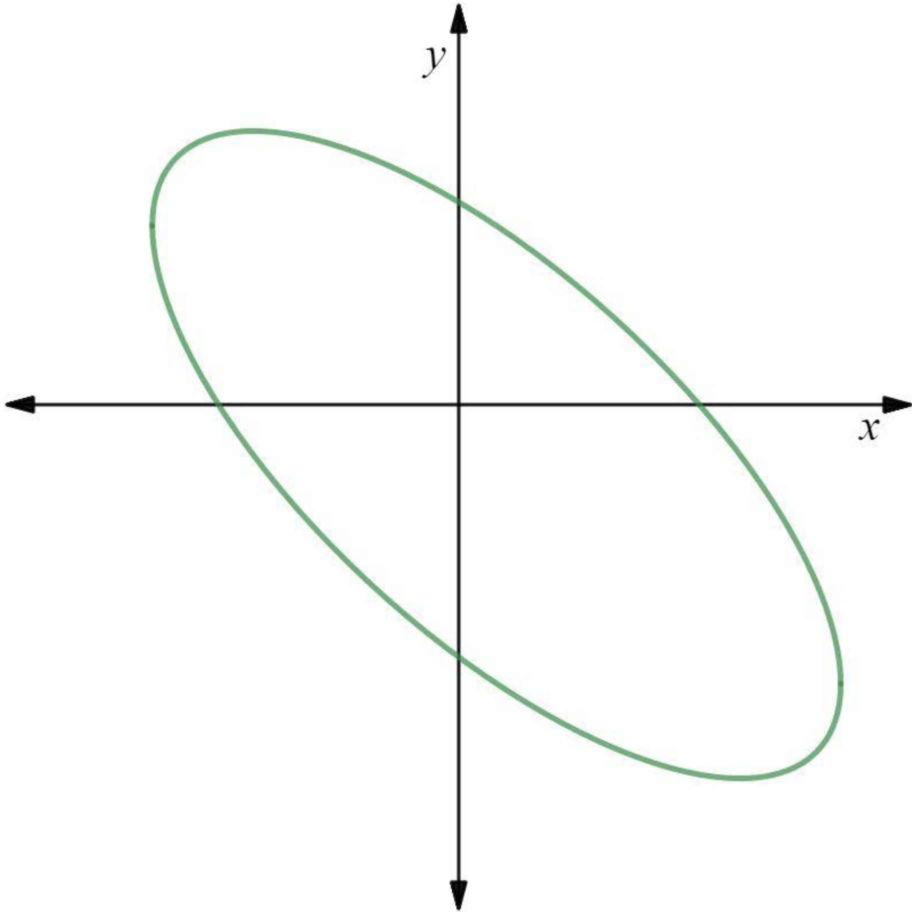
- To **differentiate** a **function** of x and y use the **product rule**.

$$(u \times v)' = u' \times v + u \times v'$$

$$\frac{d}{dx}(2xy)$$

Implicit Differentiation

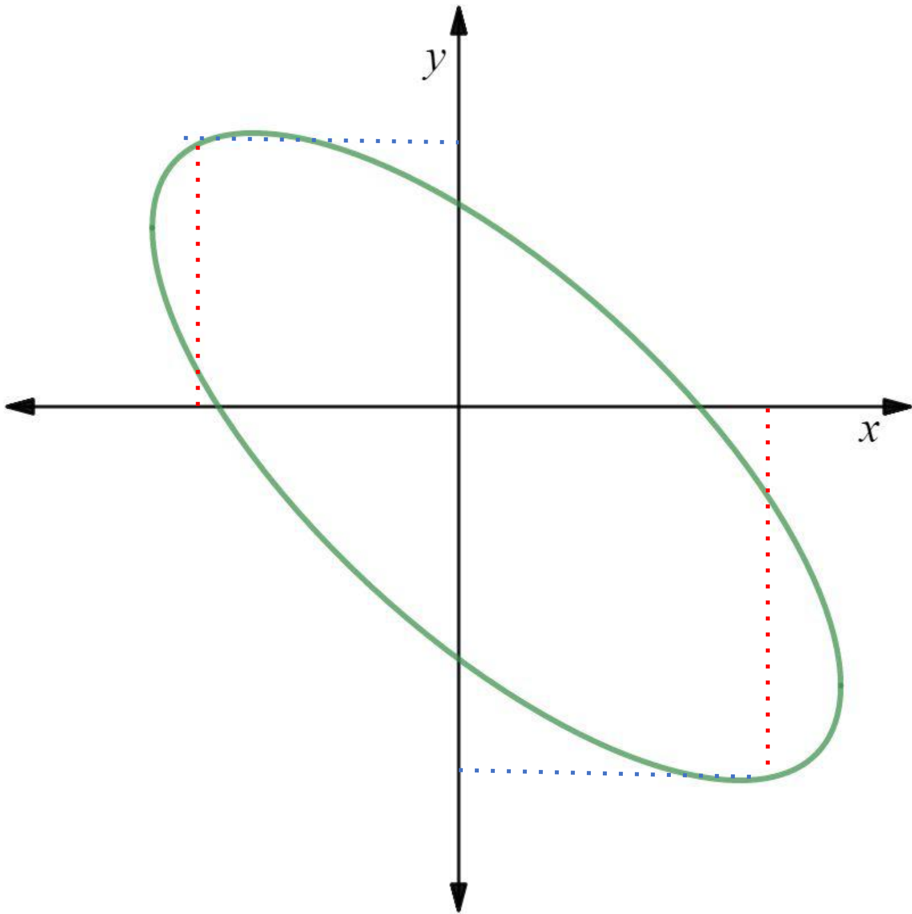
$$\frac{d}{dx}(x^2) + \frac{d}{dx}(2xy) + \frac{d}{dx}(2y^2) = \frac{d}{dx}(4)$$



Implicit Differentiation

$$\frac{dy}{dx} =$$

$$x^2 + 2xy + 2y^2 = 4$$



When will we **use** these **coordinates?**

Exemplar Exam Question

1) A curve is defined by the following equation. Find the gradient of the tangent to the curve at the point $(-1, 1)$.

$$x^2 - y + x^3y^2 + 1 = 0$$

[4 marks]

Recall formula for **implicit differentiation**.

Recall gradient of **tangent** to **curve** in **terms of** $\frac{dy}{dx}$.

Exemplar Exam Question Answer

Differentiate the equation for the curve implicitly.

$$x^2 - y + x^3y^2 + 1 = 0$$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(y) + \frac{d}{dx}(x^3y^2) + \frac{d}{dx}(1) = 0$$

$$2x - \frac{dy}{dx} + \frac{d}{dx}(x^3y^2) + 0 = 0$$

[1 mark]

Exemplar Exam Question Answer

Use the product rule to determine $\frac{d}{dx}(x^3y^2)$.

$$\frac{d}{dx}(x^3y^2) = \frac{d}{dx}(x^3)y^2 + x^3\frac{d}{dx}(y^2)$$

$$= 3x^2y^2 + 2x^3y\frac{dy}{dx}$$

[1 mark]

Exemplar Exam Question Answer

Rearrange differentiated expression to determine $\frac{dy}{dx}$.

$$2x - \frac{dy}{dx} + \left(3x^2y^2 + 2x^3y\frac{dy}{dx}\right) = 0$$

$$2x + 3x^2y^2 + (2x^3y - 1)\frac{dy}{dx} = 0$$

$$2x + 3x^2y^2 = (1 - 2x^3y)\frac{dy}{dx}$$

[1 mark]

Exemplar Exam Question Answer

$$2x + 3x^2y^2 = (1 - 2x^3y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x + 3x^2y^2}{1 - 2x^3y}$$

Determine $\frac{dy}{dx}$ at the point $(-1, 1)$.

$$\frac{dy}{dx} = \frac{2(-1)^2 + 3(-1)^2(1)^2}{1 - 2(-1)^3(1)} = \frac{2+3}{1+2} = \frac{5}{3}$$

[1 mark]

MINI MOCK PAPER

Exam Question

1. Determine $\frac{dy}{dx}$ for the following expression.

$$y = \frac{x^{\frac{3}{2}}}{\tan x}$$

[3 marks]

Exam Question Answer

1. Apply quotient rule

$$u = x^{\frac{3}{2}}, \qquad u' = \frac{3}{2}x^{\frac{1}{2}}$$

$$v = \tan x, \qquad v' = \sec^2 x$$

[1 marks]

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$$

$$= \frac{\frac{3}{2}x^{\frac{1}{2}}\tan x - x^{\frac{3}{2}}\sec^2 x}{\tan^2 x} = \frac{3}{2}x^{\frac{1}{2}}\cot x - x^{\frac{3}{2}}\operatorname{cosec}^2 x$$

[2 marks]

Exam Question

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2. Use implicit differentiation to determine $\frac{dy}{dx}$ for the following expression in terms of x.

$$x = \sin y$$

[3 marks]

Exam Question Answer

2. Apply implicit differentiation

$$1 = \cos y \times \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

[1 mark]

Write $\cos y$ in terms of x

Recall $\sin^2 y + \cos^2 y = 1$

$$\Rightarrow \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

[1 mark]

Exam Question Answer

Substitute into differential

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

[1 mark]