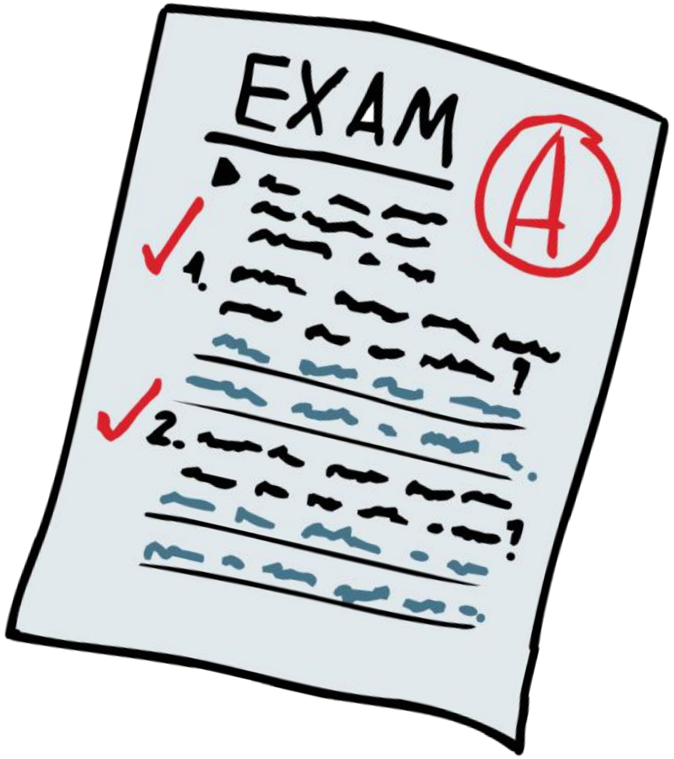
Exam Technique

What are we going to cover?

- **Interpreting questions**
- How to **set out answers**
- Tried and tested **exam wisdom**

Past Papers



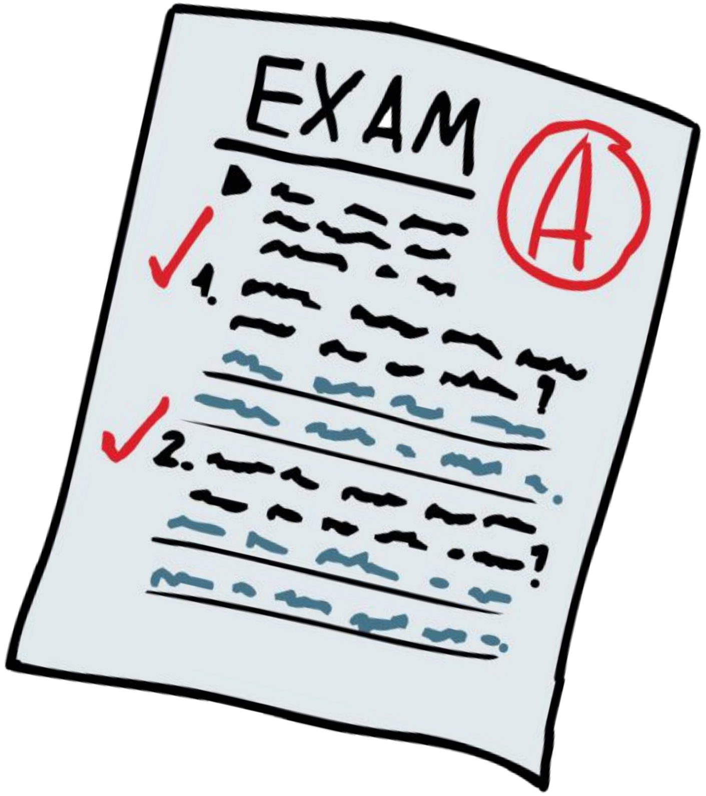
Identify **which topics** appear in **which papers** for your exam board.

All the **specifications changed** in 2017 so there is a very limited number of papers for the new specifications (**including released mock and optional papers**): you can look at the old-style ones but be aware that there are a number of **changes.**

Past Papers

The newer papers that you'll take have:

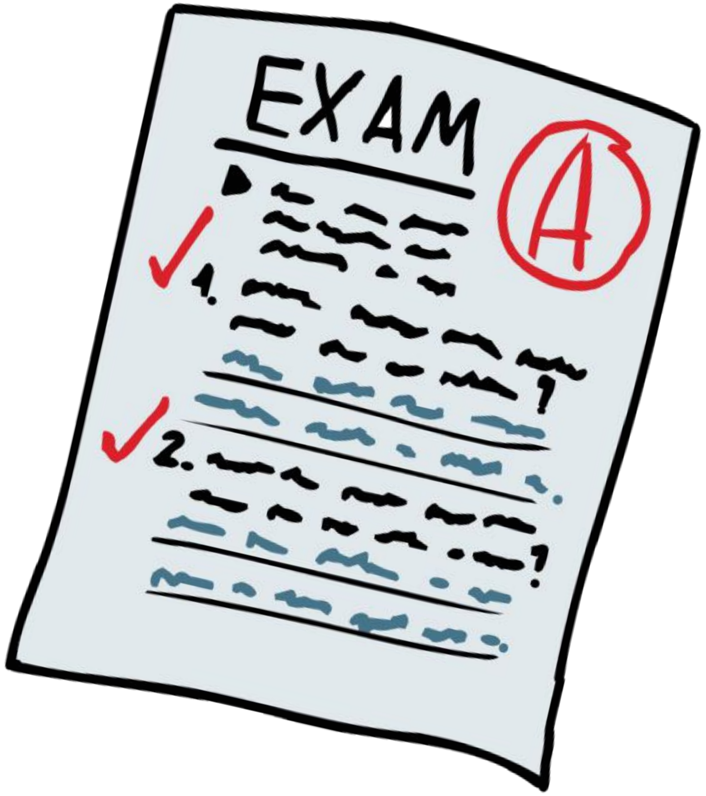
- **More** questions that link **different topics** in the specification.
- **More** questions involving **explanations**.
- **More modelling** questions – can you apply your maths to a new situation?
- **More unstructured** calculations – where you're not taken through step by step.



Past Papers

The newer papers that you'll take:

- **Assume knowledge of large data set** – if you have practiced using it, you will find the exam questions involving large sets of data easier.
- **Have no** decision maths.
- **Have no** modules - meaning fewer papers, but each **cover more** content.



What do the examiners advise?

“Students should be encouraged in multi-part questions to **attempt all parts**, as often marks can be picked up despite not making progress in earlier parts. “

“It is expected that learners will **simplify** algebraic and numerical expressions when giving their **final answers**, even if the question does **not** explicitly ask them to do so.”

“Final answers should be given to **3 significant figures** (unless otherwise stated) – more accurate answers will be penalised”

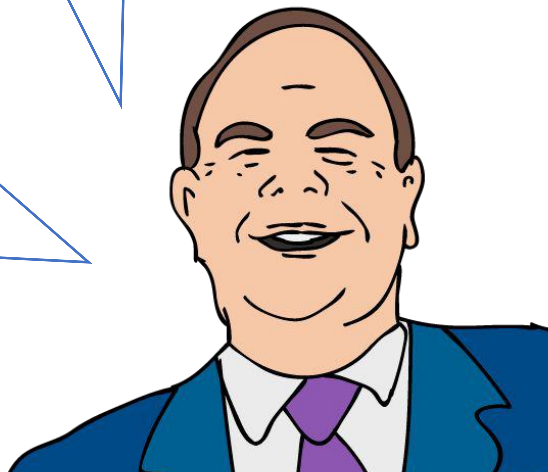
"Correct answers without working may not score **all, or indeed, any** of the marks available"

To gain full marks:

1) Write down any expressions you are going to calculate

2) Write down the values of any variables you are using

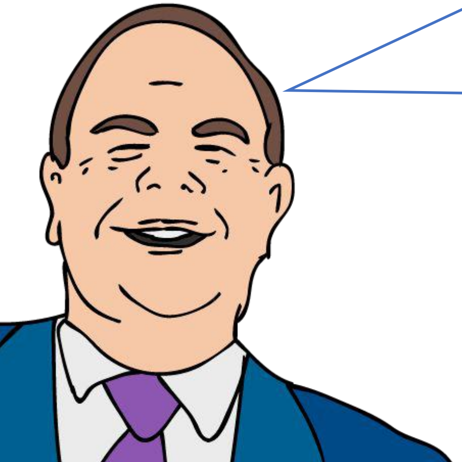
3) Use correct mathematical (**not** calculator) notation



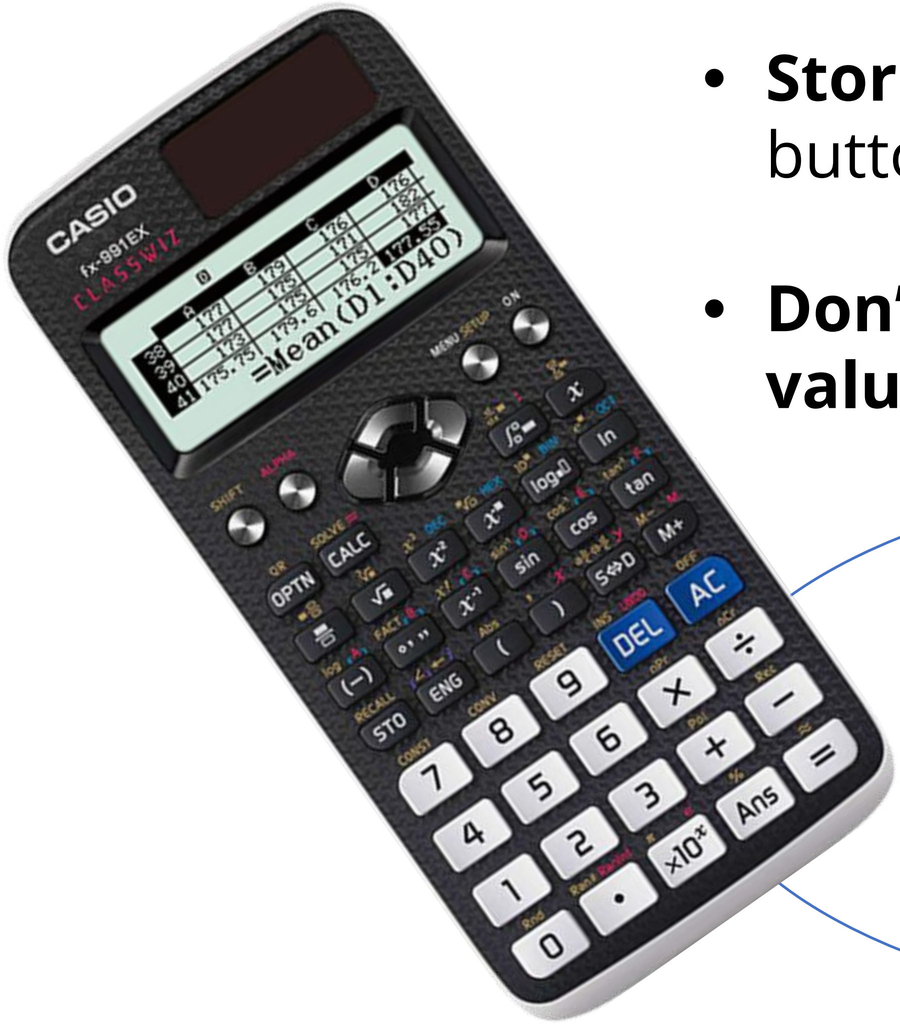
Calculator Skills

“Allowed calculator technology was used well by some students, but most **missed opportunities** to reduce the amount of routine manipulation required.”

“Students should feel **confident** that they will not be penalised for **solving a quadratic equation** on their calculators, unless full justification is **explicitly** requested.”



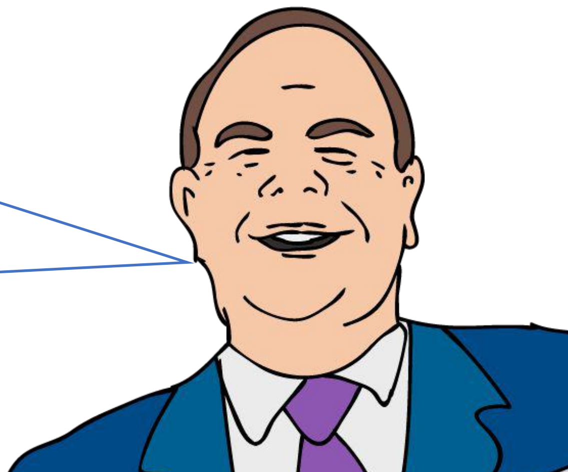
Calculator Skills



- **Store your answer** in the calculator: "**Ans**" button will use the most recent result

- **Don't round** until the **final answer** – use **exact values** throughout your working.

"Students need to **take care** when using a calculator… it is very difficult to award a mark for their method if no **working shown**"



Specific Calculator Skills for Pure Maths

- **Sketch** the **inverse** of a function e.g. $f(x) = x^3$ and $f^{-1}(x)$
- Sketch **modulus functions** e.g. $y = |2x - 3|$
- Sketch a curve given by **parametric equations** e.g. $x = 2t\ ,\ \ y = 5 - t^2$
- Calculate **terms** of a **sequence** defined by a **recurrence relation**

  e.g. $u_{n+1} = 2u_n + 3\ ,\ \ u_1 = 5$ (use the **Ans** key to access higher terms)
- Calculate the **sum of a sequence** e.g. $\sum_{x=2}^{7}(8x - 3)$

Specific Calculator Skills for Pure Maths

- Evaluate **derivatives** at a **given point** e.g. $\boldsymbol{y = x + 3\mathrm{e}^{2x}} \rightarrow \boldsymbol{\frac{\mathrm{d}y}{\mathrm{d}x}(x = 0)}$
- Find the **area** between two curves e.g. $\boldsymbol{y = x^2 - 9}$, $\boldsymbol{y = -x^2 + 8x - 9}$
- Check the **coefficients** of a **binomial expansion**

  e.g. find the $\mathbf{5}^{\text{th}}$ term of $\boldsymbol{(x - 2y)^9}$

- Check a **solution** for a set of **partial fractions** by **equating coefficients**

  e.g. $\boldsymbol{11x^2 + 14x + 5 = A(x+1)(2x+1) + B(2x+1) + C(x+1)^2}$

Specific Calculator Skills for Mechanics

- Solve **quadratic equations** involving **trigonometry**

  e.g. $\mathbf{19.62\tan^2\theta - 100\tan\theta + 39.62 = 0}$

- Find the **absolute value** of a **vector** and the **angle between two vectors**

  e.g. $\begin{pmatrix}\mathbf{6}\\\mathbf{7}\end{pmatrix}, \begin{pmatrix}\mathbf{1}\\\mathbf{0}\end{pmatrix}$

- Perform calculations on $\mathbf{3.D}$ vectors

  e.g. $\mathbf{4\vec{a} + 5\vec{b}}$

- **Change base** to **work** in **hours** and **minutes**

  e.g. $\mathbf{1.21\ hours = 1\ hour, 12\ minutes, 36\ seconds}$

Specific Calculator Skills for Statistics

- Find the **variance** of a **population**
- Find probabilities from a **Normal distribution**

  e.g. $\boldsymbol{P(4 \leq X \leq 5.5)}$ where $\boldsymbol{X \sim N\left(3, 1.5^2\right)}$

- **Inverse Normal distribution** e.g. Find $\boldsymbol{a}$ when $\boldsymbol{P(X \leq a) = 0.69}$, $\boldsymbol{X \sim N\left(11, 4^2\right)}$
- **Perform a z-test** e.g. $\boldsymbol{Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}}$ compared to $\boldsymbol{\Phi_z = P(Z \leq z)}$, $\boldsymbol{Z \sim N\left(0, 1^2\right)}$

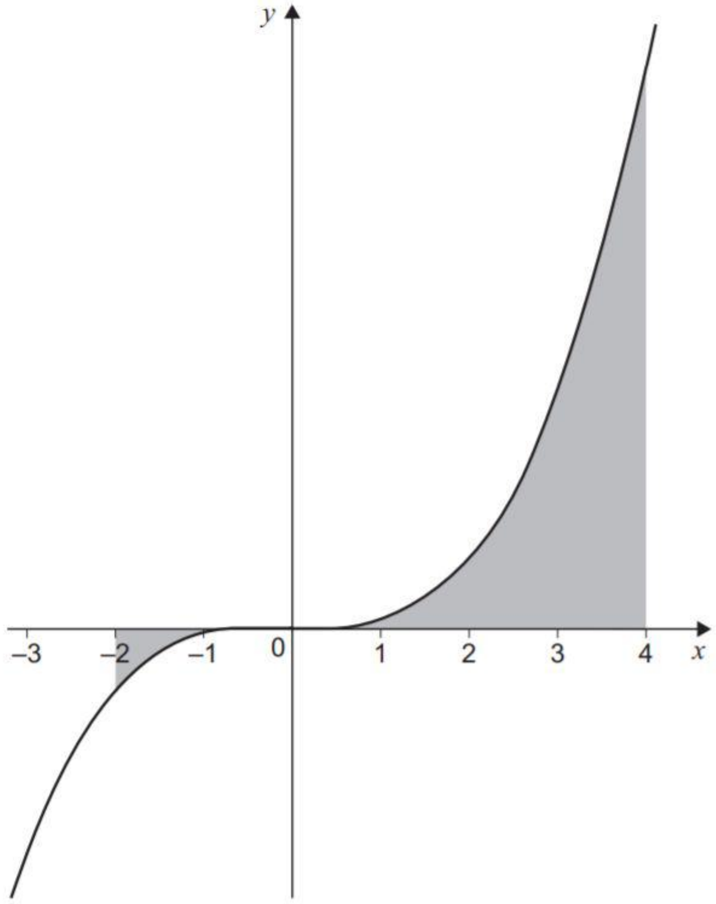
Calculator Skills

Integrate on your calculator to save time!

Here, **remember** to **split** into two separate **integrations**.

$$\left|\int_{-2}^{0} x^3\,\mathrm{d}x\right| + \left|\int_{0}^{4} x^3\,\mathrm{d}x\right| = 4 + 64 = 68$$

3 The graph of $y = x^3$ is shown.



Find the total shaded area.

[1 mark]

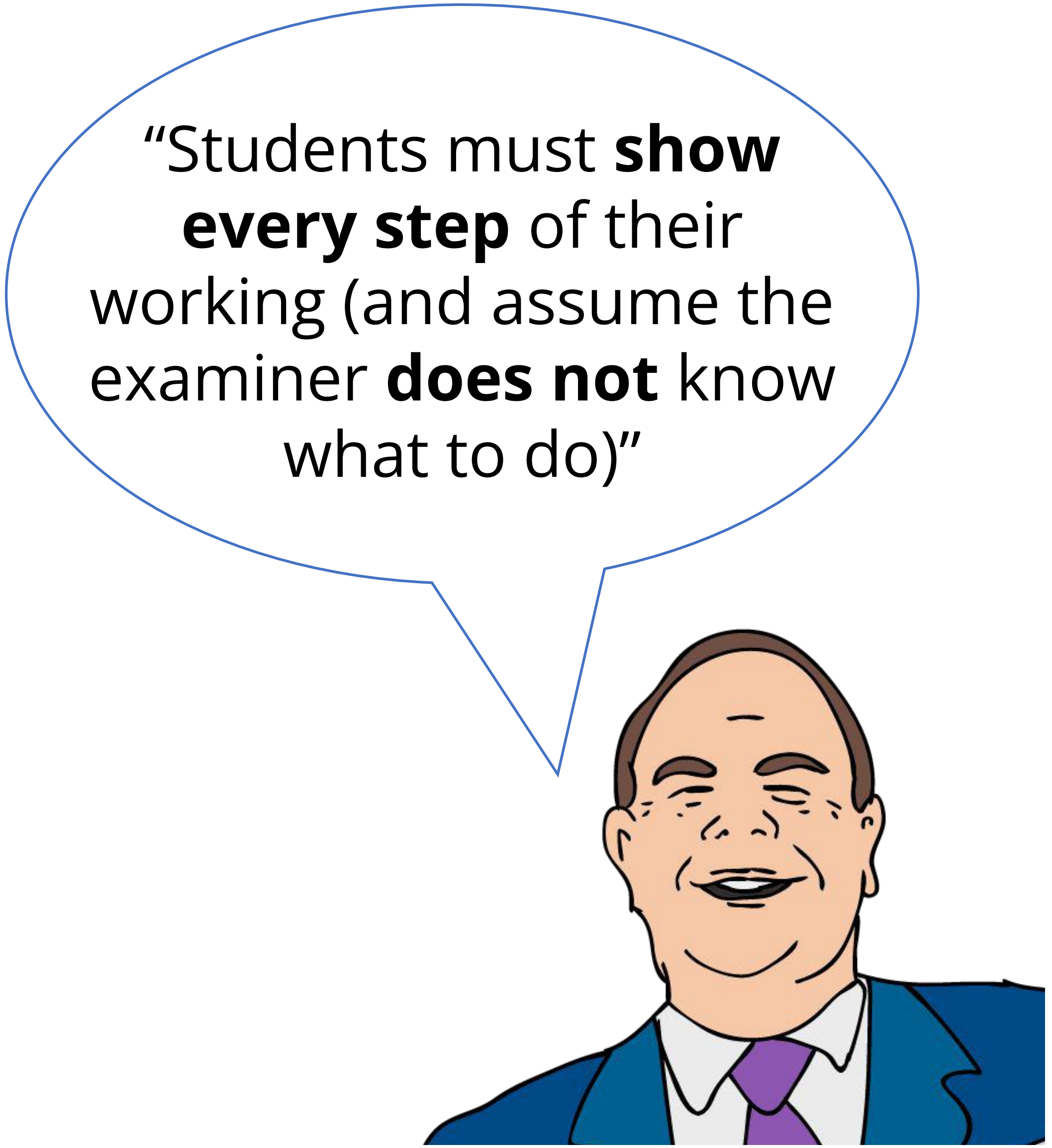
Command Words

These tell you what **form** you must write your answer in:

- **Find**
- **Calculate**
- **Solve**
- **Determine**
- **Give**
- **State**
- **Write down**
- **Prove**
- **Show that**
- **Plot**
- **Sketch**
- **Draw**
- **Explain why**
- **Justify**
- **Give a reason for**
- **Comment on**

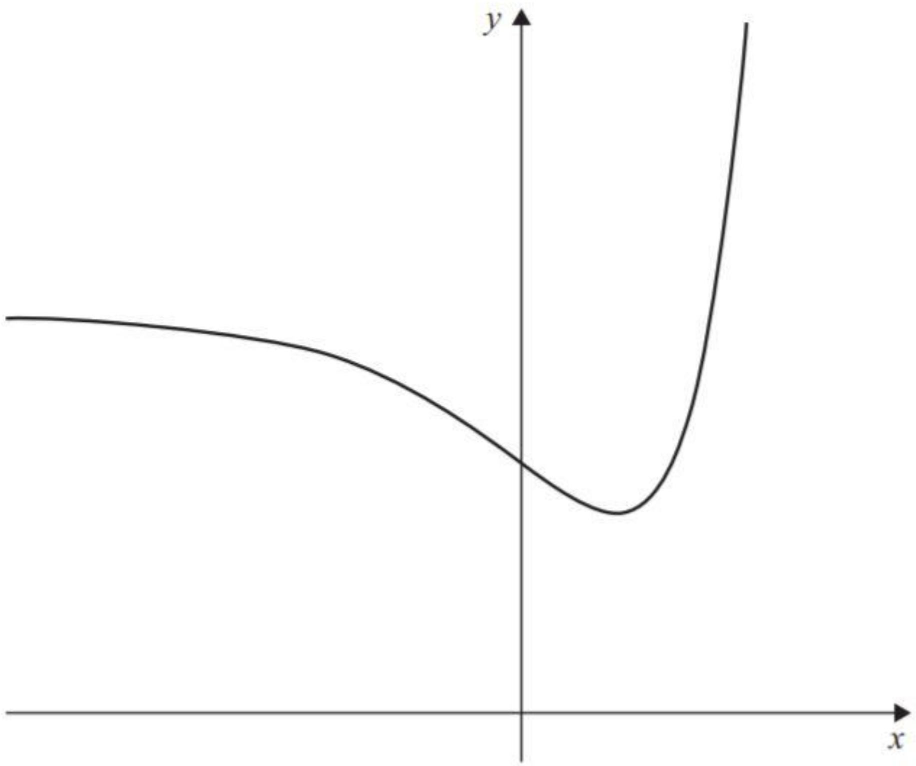
Command Words

- **Determine/ Show detailed reasoning… / Fully justify your answer/ Explain your answer**
  - Working required
  - In this question you must show **detailed reasoning / fully justify** your answer
  - Pretend the examiner does not know how to do the method!



7 A function f has domain $\mathbb{R}$ and range $\{y \in \mathbb{R} : y \geq \mathrm{e}\}$

The graph of $y = \mathrm{f}(x)$ is shown.



The gradient of the curve at the point (x, y) is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x - 1)\mathrm{e}^x$

Find an expression for $\mathrm{f}(x)$.

Fully justify your answer.

[8 marks]

Integrate by parts:

$$y = \int (x-1)e^x \, dx$$

$$\int u \frac{dv}{dx} dx = uv - \int \frac{du}{dv} v dx$$

$$u = x - 1 \qquad \frac{du}{dx} = 1$$

$$\frac{dv}{dx} = e^x \qquad v = e^x$$

$$y = (x-1)e^x - \int e^x \, dx$$

Don't forget the **constant**!

$$y = (x-1)e^x - e^x + c$$

Determine y-value of **minimum point**:

Range $\geq$ **e** $\rightarrow$ $y =$ **e at min**

Find x **coordinate** of **minimum**:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$
$$(x - 1)\mathrm{e}^x = 0$$
$$x = 1$$

So the **curve** passes through the **point** $(1, e)$

Use the **minimum point** to determine constant c:

$$\mathrm{e} = (1 - 1)\mathrm{e}^1 - \mathrm{e}^1 + c$$
$$c = 2\mathrm{e}$$

Write the **equation** for $f(x)$:

$$f(x) = (x - 1)\mathrm{e}^x - \mathrm{e}^x + 2\mathrm{e}$$
$$f(x) = (x - 2)\mathrm{e}^x + 2\mathrm{e}$$

Command Words

- **Show that**
  - **Answer** should be **sufficiently detailed** so it can be **followed**
  - Show <u>every step</u> of your working

- **Prove**
  - A **formal proof** requires a high level of mathematical detail
  - Clearly define variables
  - Include a **concise conclusion**

4. (i) Show that $\sum_{r=1}^{16}(3 + 5r + 2^r) = 131\,798$ **(4)**

Split up the sum:

$$\sum_{r=1}^{16}(3+5r+2^r) = \sum_{r=1}^{16}(3+5r) + \sum_{r=1}^{16}(2^r)$$

Calculate each part individually:

$$= \left[\frac{n}{2}(2a+(n-1)d)\right] + \left[\frac{a(1-d^n)}{1-d}\right]$$

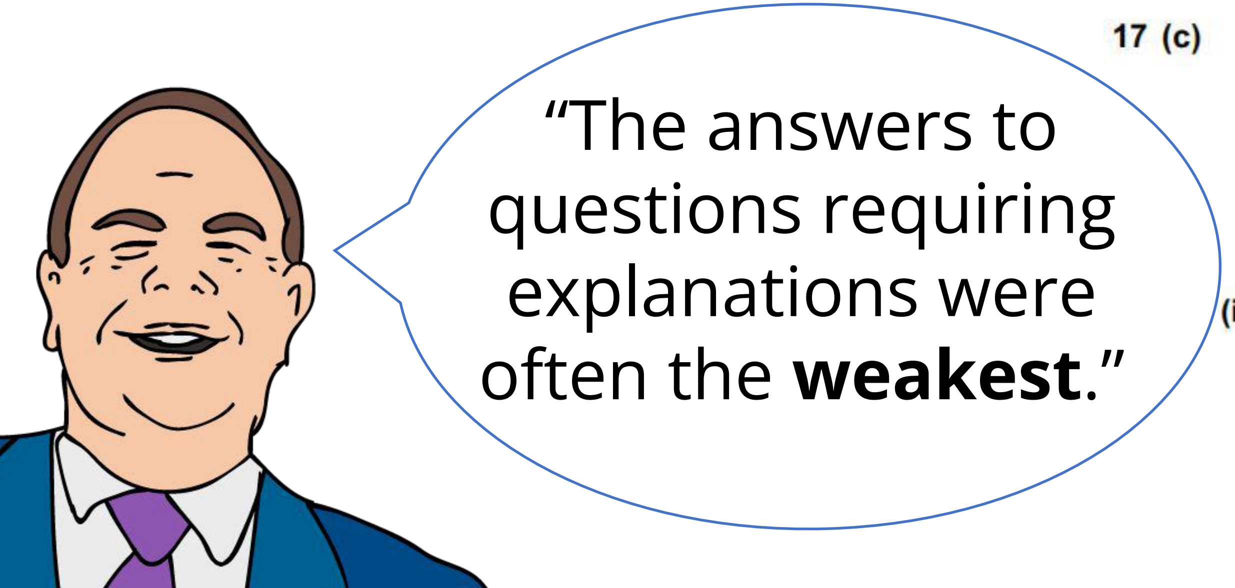
$$= \frac{16}{2}\big(2(8)+15(5)\big) + \frac{2(2^{16}-1)}{2-1}$$

Sum the results:

$$= 728 + 131070 = 131798$$

Command Words

- Explain **why**
- **Justify**
- Give a **reason** for
- **Comment** on



17 A buggy is pulling a roller-skater, in a straight line along a horizontal road, by means of a connecting rope as shown in the diagram.

The combined mass of the buggy and driver is 410 kg
A driving force of 300 N and a total resistance force of 140 N act on the buggy.

The mass of the roller-skater is 72 kg
A total resistance force of R newtons acts on the roller-skater.

The buggy and the roller-skater have an acceleration of $0.2\,\mathrm{m\,s^{-2}}$

17 (c) The roller-skater releases the rope at a point A, when she reaches a speed of $6\,\mathrm{m\,s^{-1}}$

She continues to move forward, experiencing the same resistance force.

The driver notices a change in motion of the buggy, and brings it to rest at a distance of 20 m from A.

(ii) Explain the change in motion that the driver noticed.

[2 marks]

What will the driver notice and **why**?

The rope is released, so there is no tension acting on the buggy

…therefore there is a higher resultant force on the buggy…

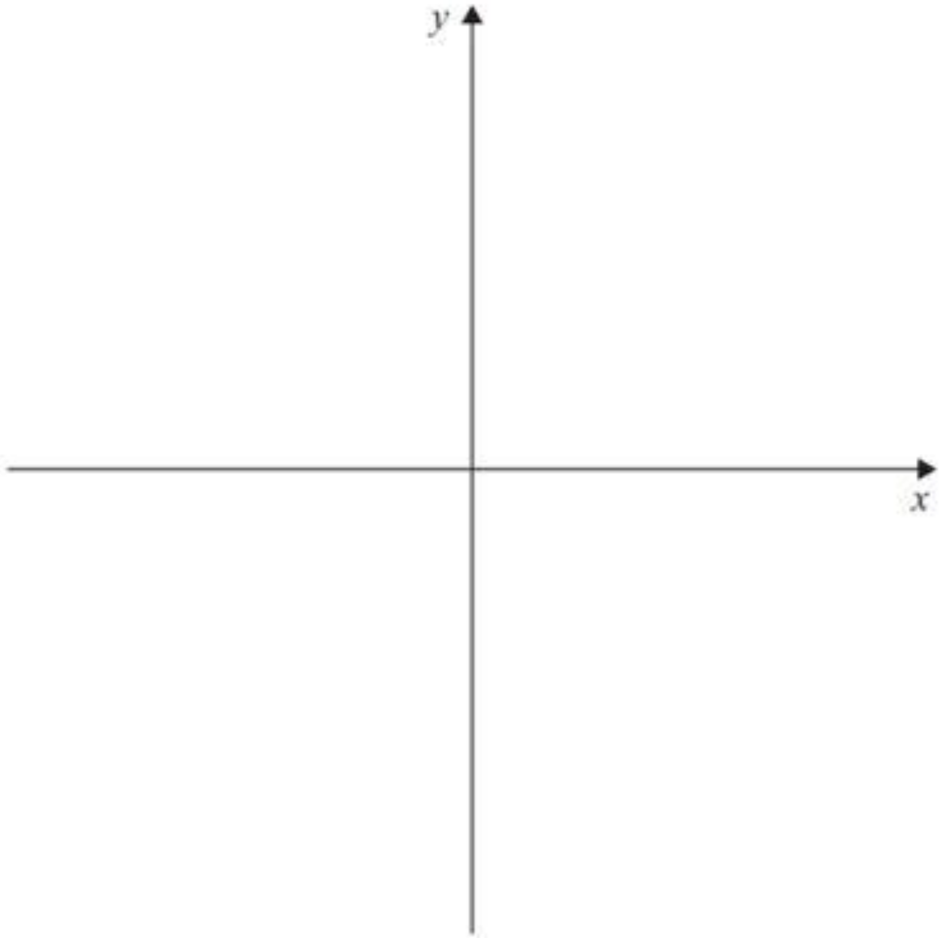
The driver will notice an increase in acceleration.

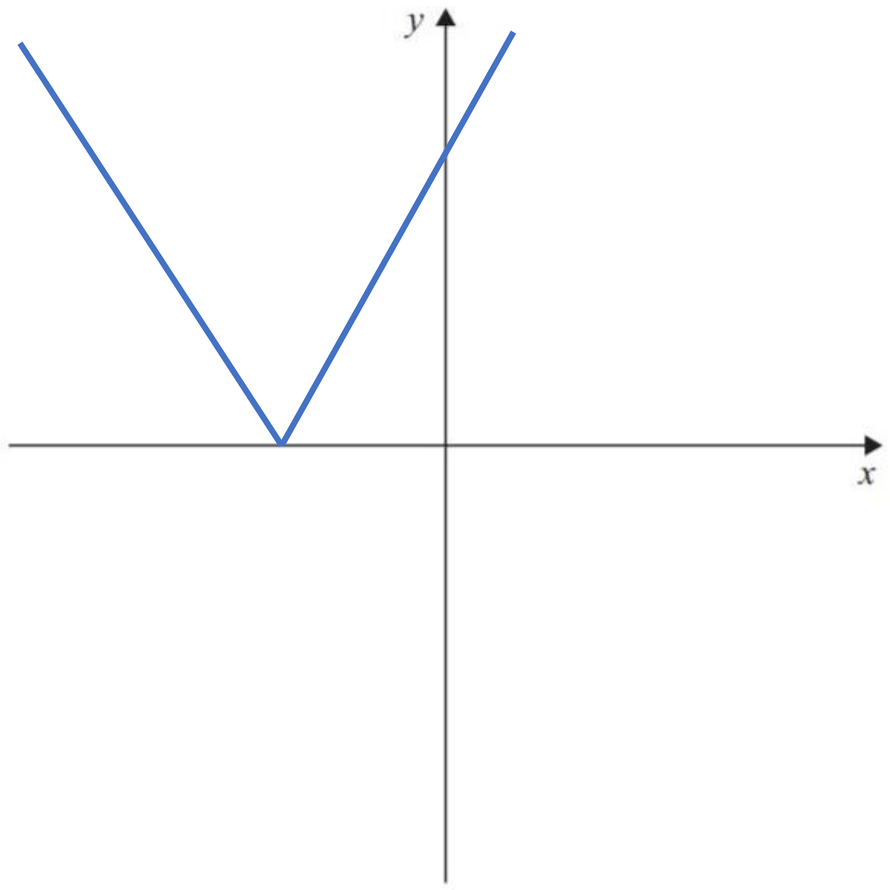
- **Plot**
  - Mark **points accurately**
- **Sketch/Draw**
  - **Not necessarily to scale**
  - **Important features** may include:
    - Turning points
    - Asymptotes
    - **Intersection** with $\boldsymbol{x}$- and $\boldsymbol{y}$-axes
    - Behaviour for **large** $\boldsymbol{x}$

Sketch the graph of $y = |2x + a|$, where a is a positive constant.

Show clearly where the graph intersects the axes.

[3 marks]





1 mark for shape only in positive quadrants

1 mark for **intercepting** $\boldsymbol{y}$-axis at $\boldsymbol{a}$

1 mark for touching $\boldsymbol{x}$-axis at $-\dfrac{\boldsymbol{a}}{\boldsymbol{2}}$

Top Tips

Look for **hints** in the question!

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

Use the **small angle approximations**:

$$\sin 3\theta \approx 3\theta \text{ and } \cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$$

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta} = \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$$

Simplify:

$$= \frac{\frac{16\theta^2}{2}}{6\theta^2}$$

$$= \frac{4}{3}$$

Top Tips

Check the **format** of your answer.

Do you need **units**?

8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(a) 6.30 am = 6.5 hours after midnight

$$\boldsymbol{D = 5 + 2\sin(30 \times 6.5)^\circ = 4.48\ \mathrm{m}}$$

(b) Minimum depth $\boldsymbol{D = 3.8\mathrm{m}}$

$$\boldsymbol{3.8 = 5 + 2\sin(30t)^\circ}$$

$$\boldsymbol{\sin(30t)^\circ = -0.6}$$

$$\boldsymbol{30t = \sin^{-1}(-0.6)}$$

$$30t = \sin^{-1}(-0.6)$$

$\sin^{-1}(-0.6)$ has many potential values, due to periodicity and symmetry of sine graph.

Know $t > 8.5$ since the boat will be loading cargo until 8.30 am, so can deduce that $30t > 30 \times 8.5 = 255$

$$30t = \sin^{-1}(-0.6) = -36.9, 217, 323$$

$$t = 10.77$$

Convert to a time:

$$10{:}46 \text{ am}$$

Know what you need to know

Key point: don't learn things if you don't have to

Edexcel

Integration (+ constant)

$\mathbf{f(x)}$	$\int \mathbf{f(x)\,dx}$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln\lvert\operatorname{cosec} kx+\cot kx\rvert$, $\quad \frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx+\tan kx\rvert$, $\quad \frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx+\frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

OCR A & B

Integration

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)|+c$$

$$\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1}+c$$

Integration by parts $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$

AQA

Integration

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)|+c$$

$\mathbf{f(x)}$	$\int \mathbf{f(x)\,dx}$
$\tan x$	$\ln\lvert\sec x\rvert + c$
$\cot x$	$\ln\lvert\sin x\rvert + c$

Learn: Trigonometric Ratios & Formulae

SOH

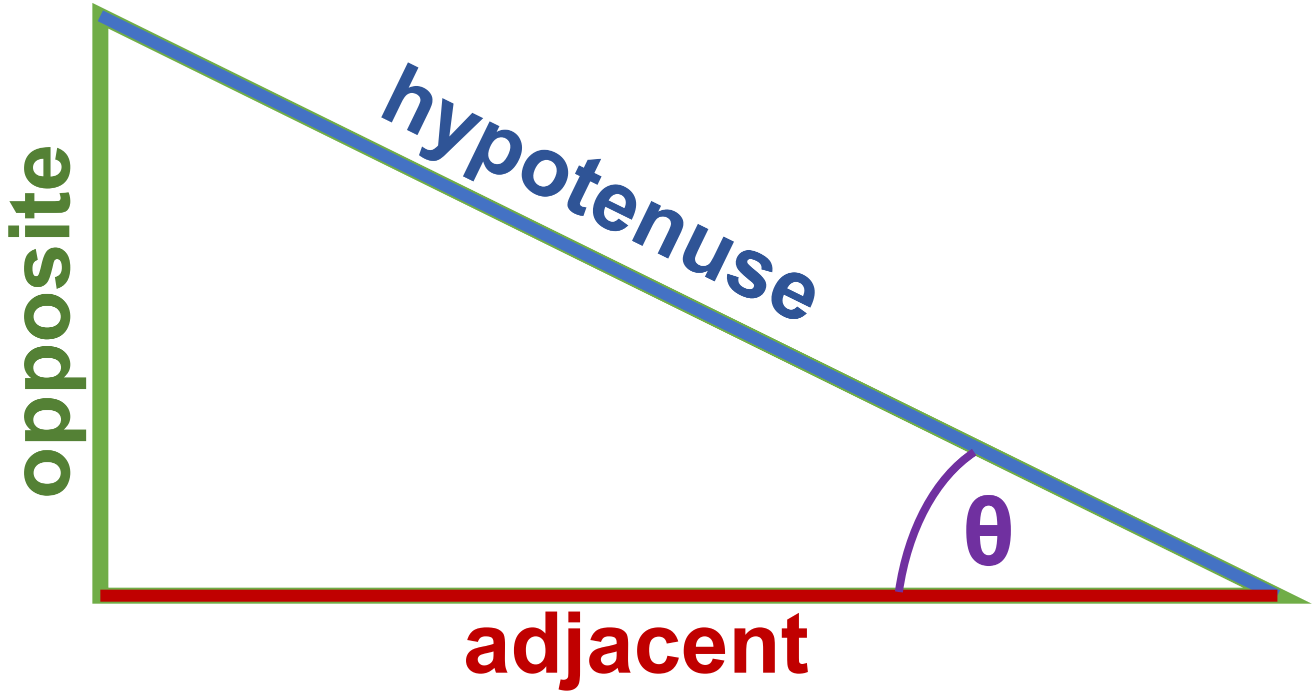
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

CAH

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

TOA

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

Top Tips for Mechanics

- Use **consistent units**.
- **Draw** a **diagram** and use it to **figure** out the **applicable method** e.g.
    - Constant acceleration **SUVAT**.
    - Constant velocity **Balancing forces**.
    - Non-linear velocity **Integration** and **differentiation**.

Large Data Set

Questions have been chosen to give an **advantage** to those who are **familiar** with the large data set. Learn:

- **Terms** and **abbreviations**
- **Uncommon units** (knots, oktas, cubic cm, physicians per 1000 population etc) may need converting
- **Limitations** on data e.g. dates, times, locations
- **Omitted or n/a** entries should not count towards totals

Notes, available online for each large data set, highlight **important features** that may be **useful to learn** for the exam