Exam Technique

Aims for today

I've chosen year 12 Maths topics that have overlap with other topics, can be used to summarise concepts and that are covered by AQA, Edexcel and OCR

Students taking A-Level Maths OR AS Maths: revise in a way that gets you thinking about answering questions in the exam

Exams this year

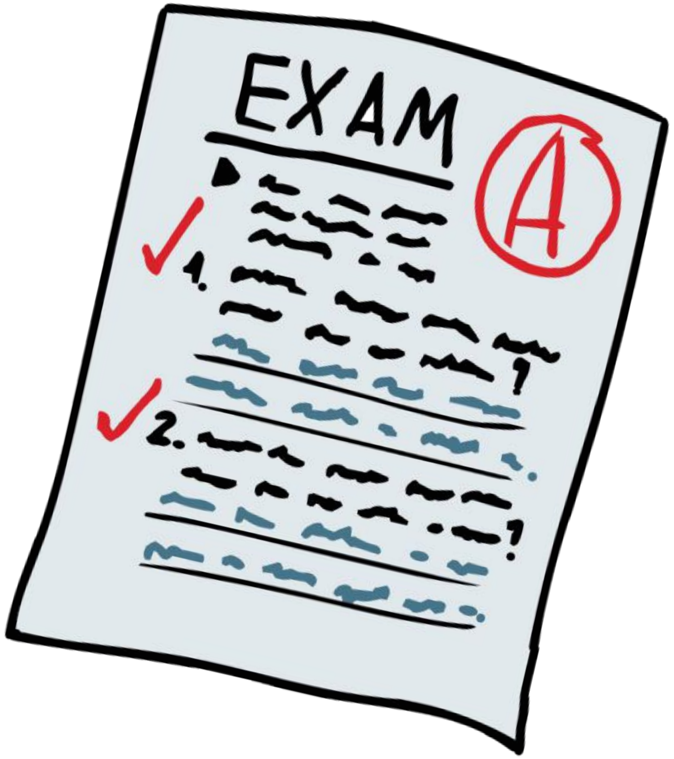
No exams this year

Year 12s taking A-Level Maths: using what you've learnt this year, maximise your UNDERSTANDING to give you a good foundation for year 13

What are we going to cover?

- **Interpreting questions**
- How to **set out answers**
- Tried and tested **exam wisdom**

Past Papers



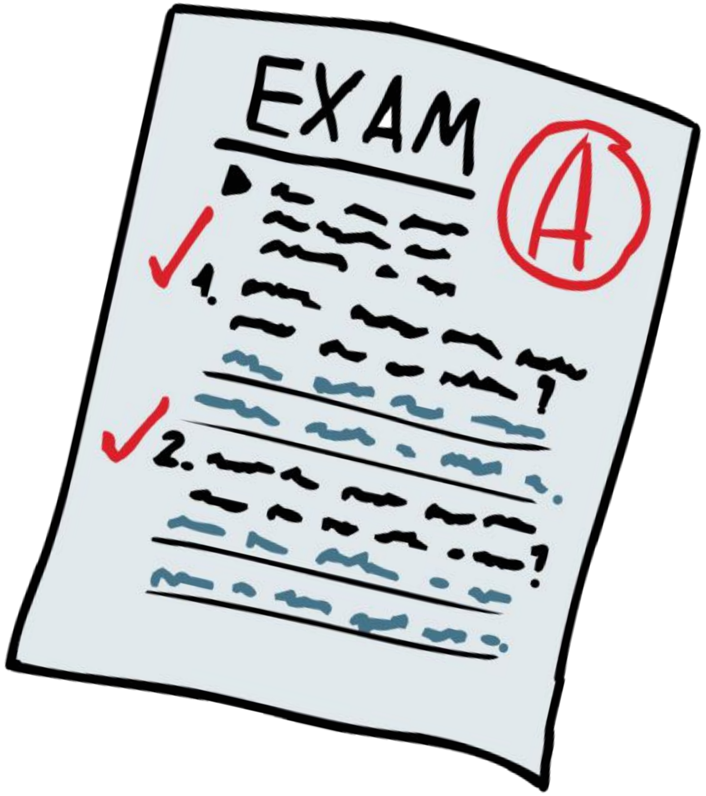
Identify **which topics** appear in **which papers** for your exam board.

All the **specifications changed** in 2017 so there is a very limited number of papers for the new specification (**including released mock and optional papers**): you can look at the old-style ones but be aware that there are a number of **changes.**

Past Papers

The newer papers that you'll take have:

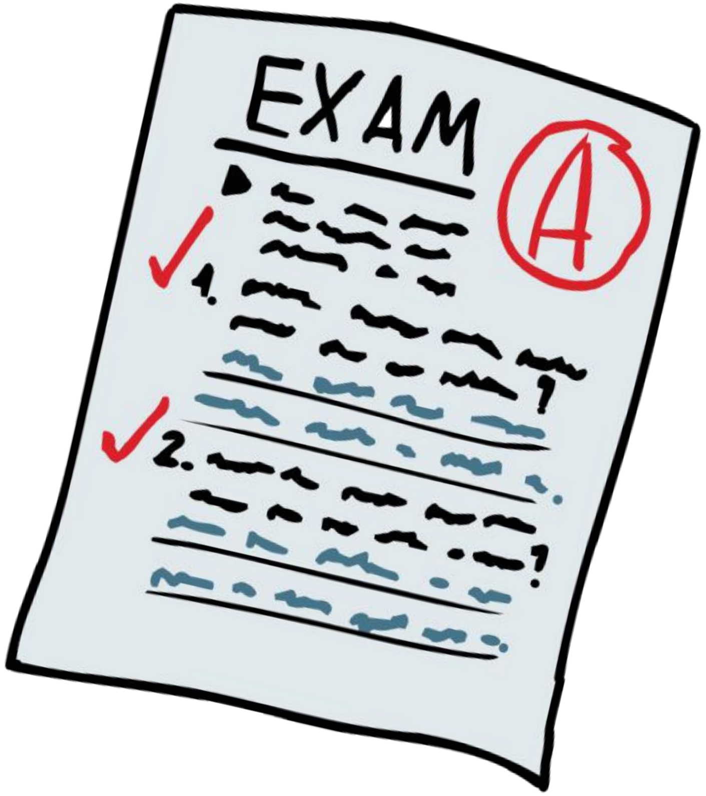
- **More** questions that link **different topics** in the specification.
- **More** questions involving **explanations**.
- **More modelling** questions – can you apply your maths to a new situation?
- **More unstructured** calculations – where you're not taken through step by step.



Past Papers

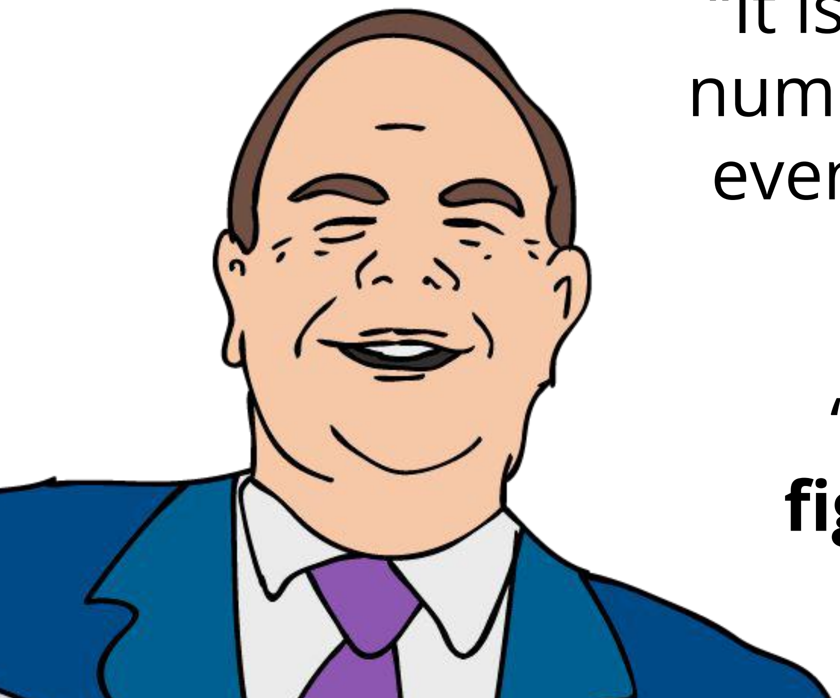
The newer papers that you'll take have:

- **Large data set** – if you have practiced using it, you will find the exam questions involving large sets of data easier.
- **No** decision maths.
- **No** modules fewer papers, but each **cover more** content.



What do the examiners advise?

"Students should be encouraged in multi-part questions to **attempt all parts**, as often marks can be picked up despite not making progress in earlier parts. "



"It is expected that learners will **simplify** algebraic and numerical expressions when giving their **final answers**, even if the question does **not** explicitly ask them to do so."

"Final answers should be given to **3 significant figures** (unless otherwise stated) – more accurate answers will be **penalised**"

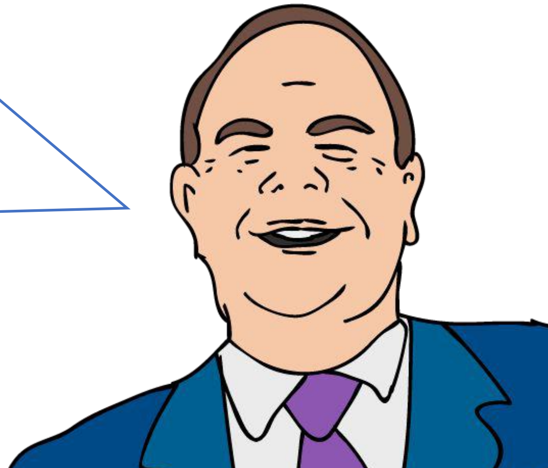
"Correct answers without working may not score **all, or indeed, any** of the marks available"

To gain **full marks**:

1) Write down **any expressions** you are going to calculate.

2) Write down the values of **any variables** you are using.

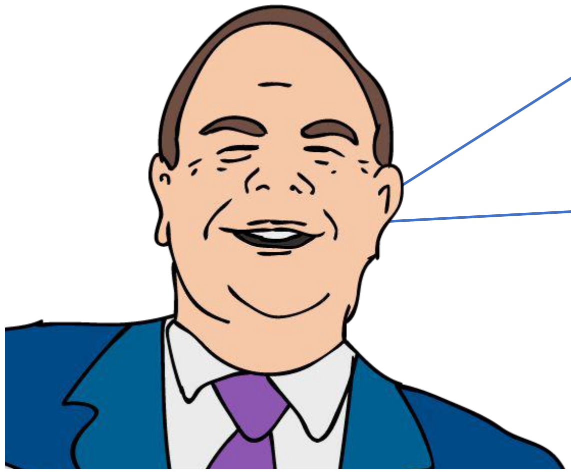
3) Use correct mathematical (**not** calculator) notation.



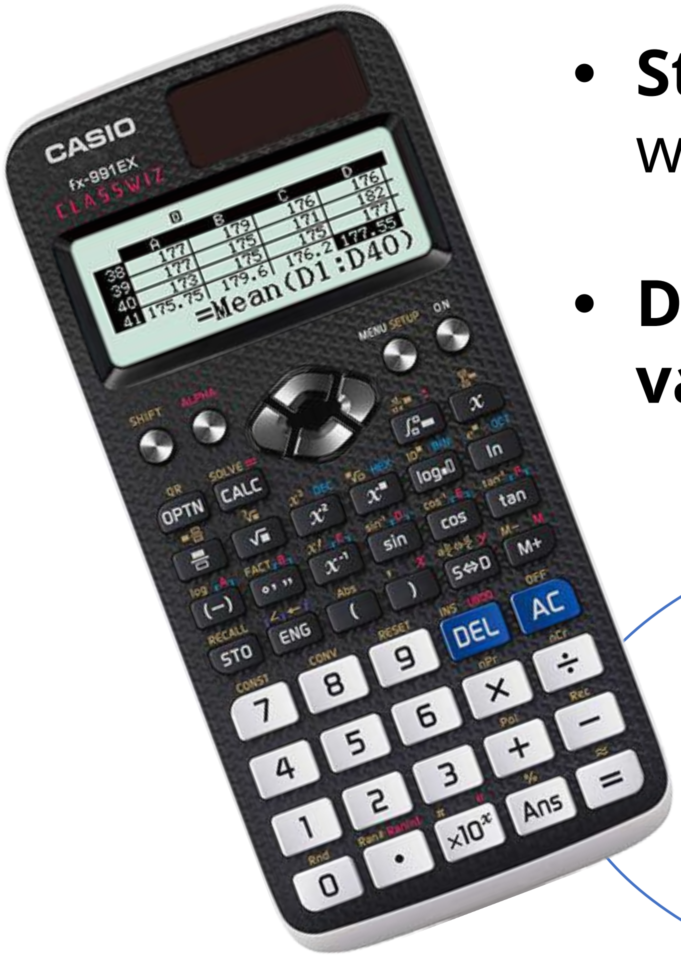
Calculator Skills

"Allowed calculator technology was used well by some students, but most **missed opportunities** to reduce the amount of routine manipulation required. "

"Students should feel **confident** that they will not be penalised for **solving a quadratic equation** on their calculators, unless full justification is **explicitly** requested."



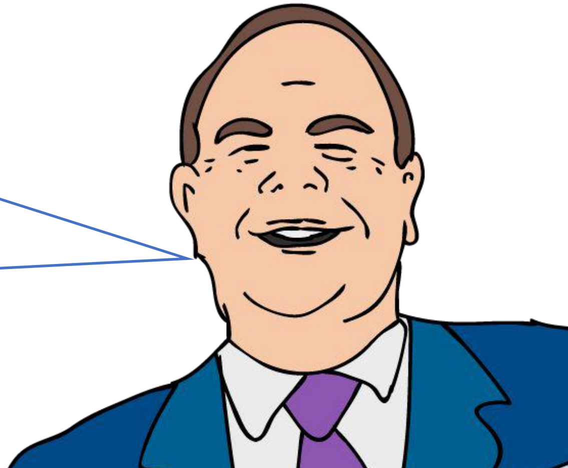
Calculator Skills



- **Store your answer** in your **calculator**: “**Ans**” button will use the most recent result.

- **Don’t round** until the **final answer** – use **exact values** throughout your working.

“Students need to **take care** when using a **calculator**… it is very difficult to award a mark for their method if **no working** is **shown**”



Specific Calculator Skills for Pure Maths

- **Solve quadratics** e.g. $2x^2 - 3x - 20 = 0$
- Evaluate the **gradient of the tangent to a curve** at a given point

  e.g. $\frac{d}{dx}(5x^2 - 2x)\Big|_{x=3}$

- **Sketch a function** and its **derivative** e.g. $f(x) = 3x^4 - x^3 + 1$ (best used as a check)
- Evaluate a **definite integral** (between two values) e.g. $\int_1^2 3\,x^4 - x^3 + 1\,dx$
- Solve equations with **exponents** e.g. $3^{2x-1} = 10$
- Calculate **factorials** using ! e.g. $5!$ and **combinations** using ${}^{n}C_r$ e.g. ${}^{5}C_2$

Specific Calculator Skills for Mechanics

- Check acceleration from **SUVAT** calculations

  e.g. $\boldsymbol{s = t^4 - 32t}$ → check that $\frac{\mathbf{d}^2\boldsymbol{s}}{\mathbf{d}\boldsymbol{t}^2}(\boldsymbol{t} = \mathbf{2})$ agrees with your answer

- Solve **simultaneous equations**

  e.g. $\boldsymbol{3g - T = 3a}$ and $\boldsymbol{T = 10 + 2a}$

- **Change base** to **work** in **hours** and **minutes**

  e.g. $\mathbf{1.21\ hours = 1\ hour, 12\ minutes, 36\ seconds}$

Specific Calculator Skills for Statistics

- Calculate the **mode**, **mean** and **median** for a set of data
- Calculate the **standard deviation** of a population (or estimate the standard deviation using a sample)
- Calculate **binomial probabilities**

  e.g. $\boldsymbol{P(X \leq 5)}$ for a binomial probability distribution with $\mathbf{30}$ trials and a probability of success in any given trial of $\mathbf{0.2}$.

Command Words

These tell you what **form** you must write your answer in:

- **Find**
- **Calculate**
- **Solve**
- **Determine**

- **Give**
- **State**
- **Write down**

- **Prove**
- **Show that**

- **Plot**
- **Sketch**
- **Draw**

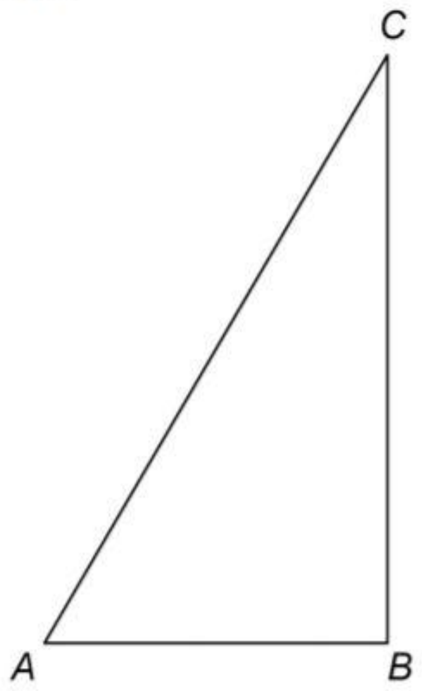
- **Explain why**
- **Justify**
- **Give a reason for**
- **Comment on**

Command Words

- **Show that**
  - Answer should be sufficiently detailed so it can be followed
  - Show <u>every step</u> of your working

- **Prove**
  - A **formal proof** requires a high level of mathematical detail
  - Clearly define variables
  - Include a **concise conclusion**

6 *ABC* is a right-angled triangle.

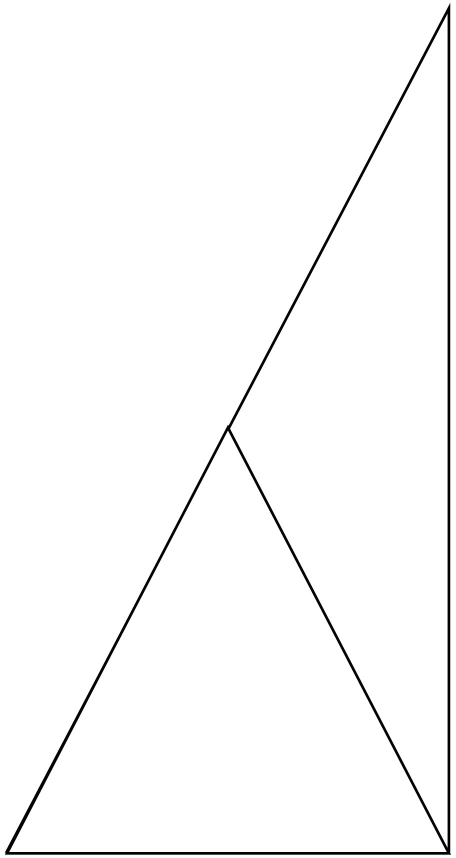


D is the point on hypotenuse *AC* such that $AD = AB$.

The area of ΔABD is equal to half that of ΔABC.

6 (a) Show that $\tan A = 2\sin A$

[4 marks]



Aim: "**Well-constructed mathematical argument** that **simplifies** to the **correct conclusion**".

$$\tan A = \frac{BC}{a}$$
$$BC = a \tan A$$

Area of ABD = $\frac{1}{2} a^2 \sin A$

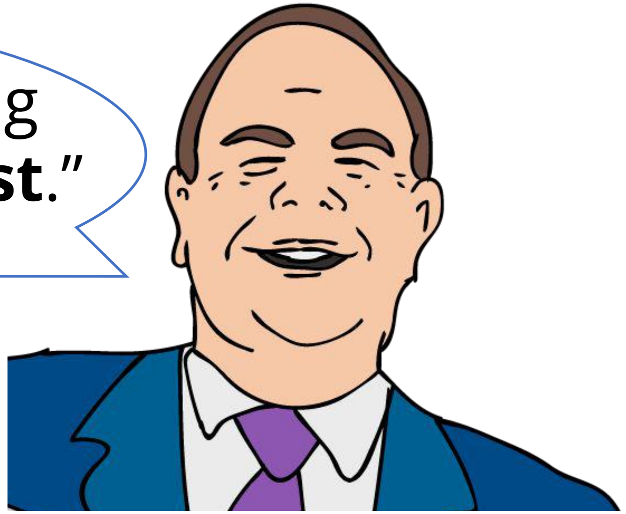
Area of ABC = $\frac{1}{2} \times a \times BC$
$= \frac{1}{2} a^2 \tan A$

Area of ABC = 2 × Area of ABD
$$\frac{1}{2} a^2 \tan A = 2 \times \frac{1}{2} a^2 \sin A$$
$$\tan A = 2 \sin A$$

Command Words

- Explain **why**
- **Justify**
- **Give** a **reason** for
- **Comment** on

"The answers to questions requiring explanations were often the **weakest**."



(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

Not always true (with a reason)

Sometimes true (with a reason)

Algebraic approach:

$$(x+3)^2 > x^2$$

$$x^2 + 6x + 9 > x^2$$

$$6x + 9 > 0$$

True for:

$$x > -\frac{9}{6} \quad \rightarrow \quad x > -\frac{3}{2}$$

→ Sometimes true

Numerical approach:

Test for $x = -4$

$$(-4+3)^2 > (-4)^2$$

$$1^2 > 4^2 \text{ not true}$$

Test for $x = +4$

$$(+4+3)^2 > (+4)^2$$

$$7^2 > 4^2 \text{ true}$$

→ Sometimes true

- **Plot**
  - Mark **points accurately**

- **Sketch/Draw**
  - **Not necessarily to scale**
  - **Important features** may include:
    - Turning points
    - Asymptotes
    - **Intersection** with x- and y-axes
    - Behaviour for **large** x

13 A vehicle, which begins at rest at point P, is travelling in a straight line.

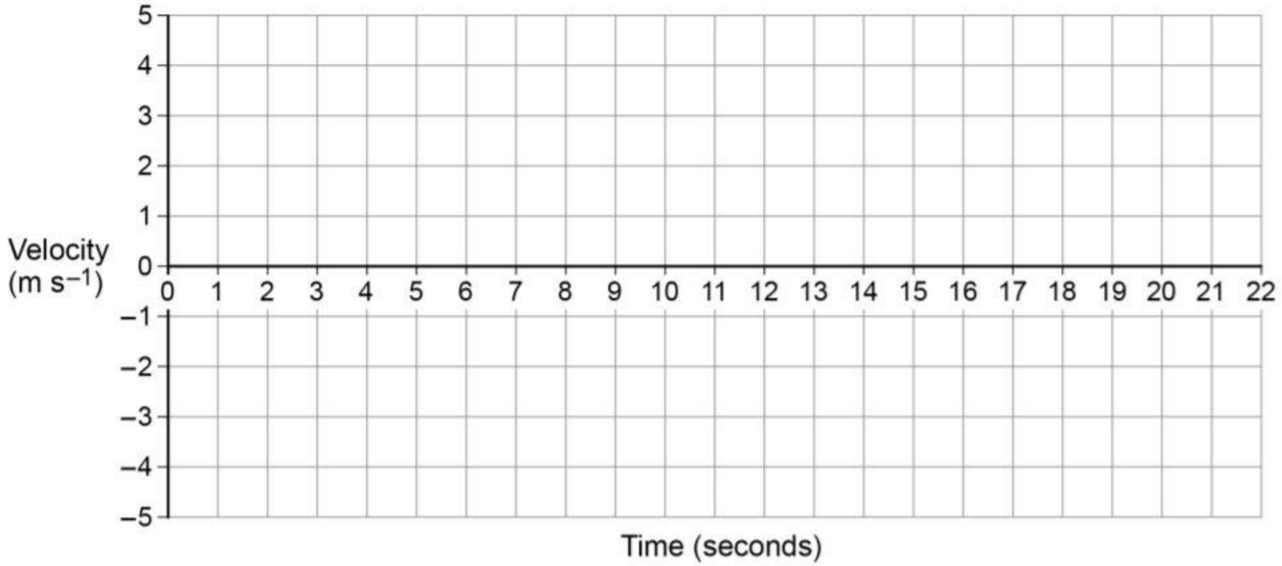
For the first 4 seconds the vehicle moves with a constant acceleration of $0.75\,\text{m s}^{-2}$

For the next 5 seconds the vehicle moves with a constant acceleration of $-1.2\,\text{m s}^{-2}$

The vehicle then immediately stops accelerating, and travels a further 33 m at constant speed.

13 (a) Draw a velocity–time graph for this journey on the grid below.

[3 marks]



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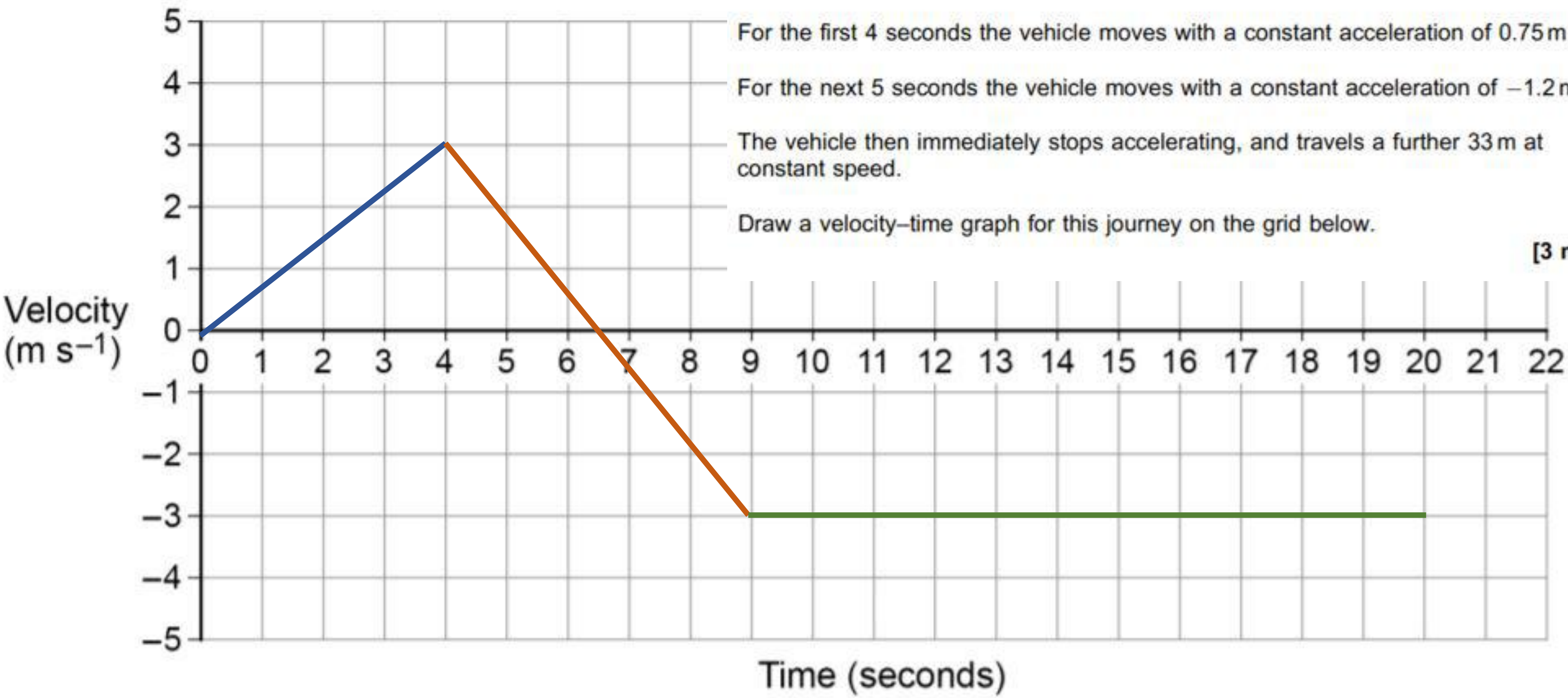
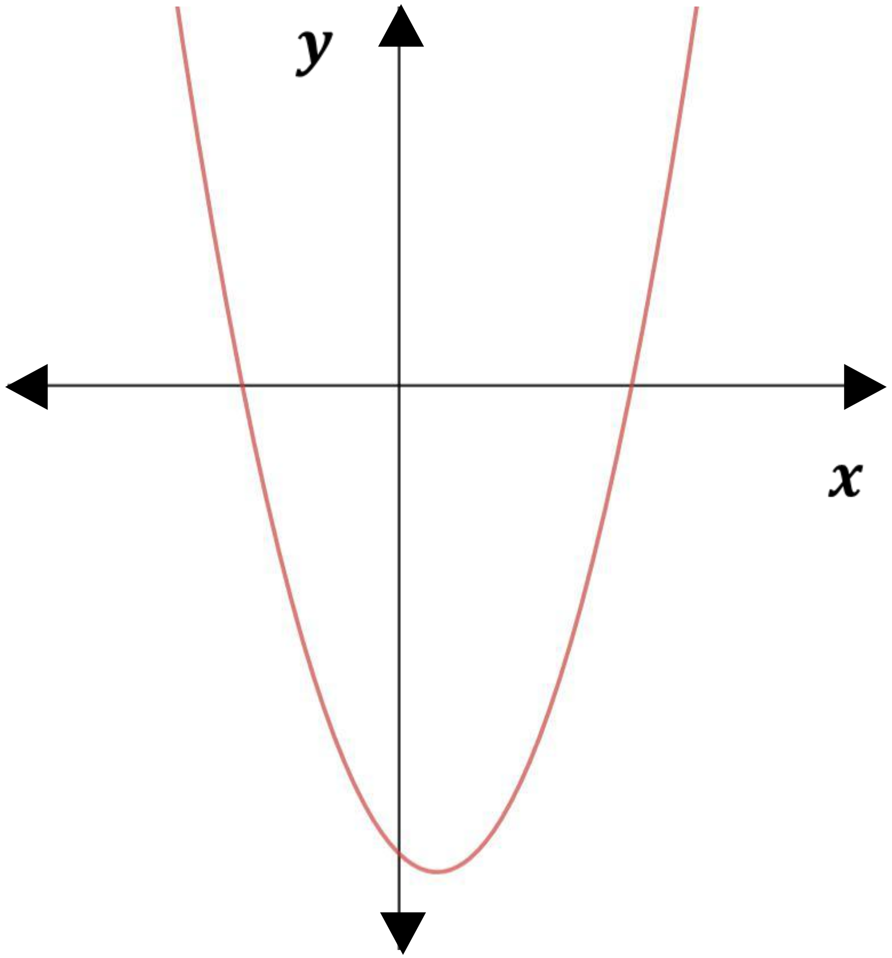


(a) Sketch the curve $y = 2x^2 - x - 3$.



Solve quadratic to give roots: $x = \frac{3}{2}, x = -1$

Curve shape:

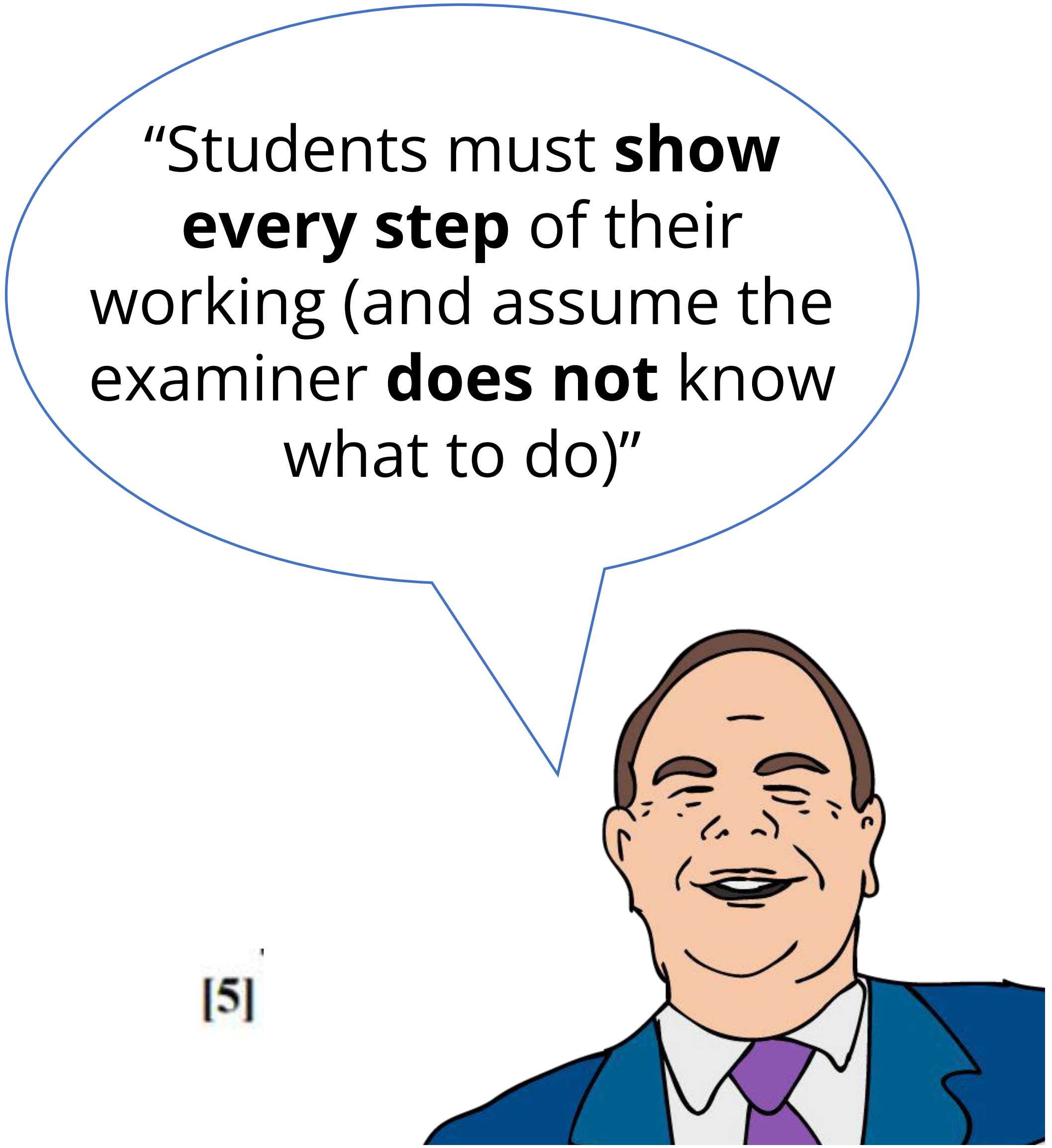
- **Symmetrical positive quadratic** U-shape.
- ***x*-intercepts** clearly labelled.
- **Minimum point** in the correct quadrant.

Command Words

- **Show detailed reasoning…**
  - Justification/ working required
  - In this question you must show detailed reasoning / fully justify your answer
  - Pretend the examiner does not know how to do the method!

2 **In this question you must show detailed reasoning.**

Solve the equation $2\cos^2 x = 2 - \sin x$ for $0^\circ \leq x \leq 180^\circ$. **[5]**



$$\cos^2 x = 1 - \sin^2 x$$

$$0 \le x \le 180°$$

$$2\cos^2 x = 2 - \sin x$$

$$2(1 - \sin^2 x) = 2 - \sin x$$

$$2 - 2\sin^2 x = 2 - \sin x$$

$$2\sin^2 x - \sin x = 0$$

$$\sin x\,(2\sin x - 1) = 0$$

or any other valid method to solve the quadratic in $\sin(x)$

$\sin x = 0$ gives $x = 30°$ or $x = 150°$

$\sin x = \frac{1}{2}$ gives $x = 0$ or $x = 180°$

Top Tips

Look for **hints** in the question!

9. $$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

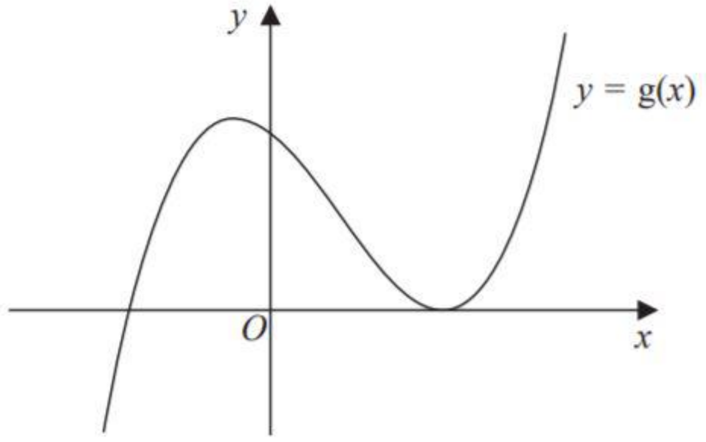


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Substitute $x = -2$ into $g(x)$:

$$g(-2) = 4 \times (-8) - 12 \times (4) - 15 \times (-2) + 50 = 0$$

Use the factor theorem!

$$g(-2) = 0 \quad \rightarrow \quad (x+2) \text{ is a factor}$$

(b) Use long division or inspection:

$$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$$

Factorise the quadratic:

$$= (x+2)(2x-5)^2$$

(c) (i) Solution is where the curve is on or below the $\boldsymbol{x}$-axis.

$$\boldsymbol{x \leq -2, x = 2.5}$$

(ii) Solution is where the curve is on the $\boldsymbol{x}$-axis so $\boldsymbol{g(2x) = 0}$

$$\boldsymbol{x = -1, x = 1.25}$$

Top Tips

Check the format of your answer.

Do you need units?

$$P > 80$$

$$100 - 6.25(x-9)^2 > 80$$

$$(x-9)^2 < 3.2$$

$$9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$$

Minimum price = £7.22

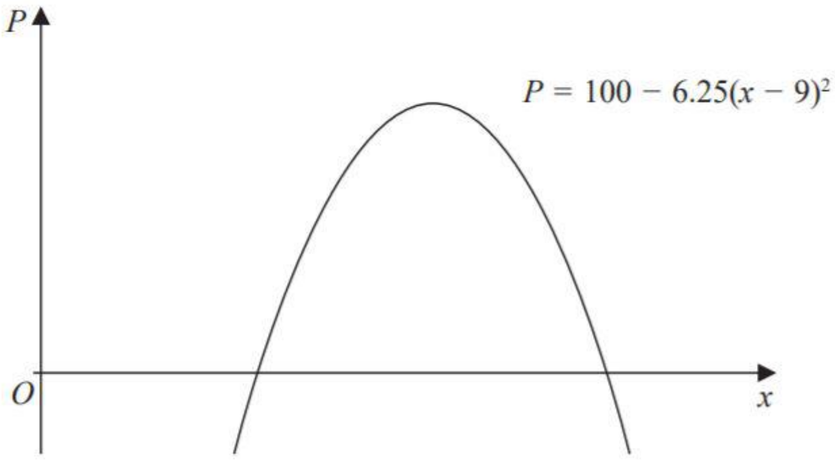


Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

Know your Formula Booklet

Key point: don't learn things if you don't have to

AQA

Integration

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$f(x)$	$\int f(x)\,dx$
$\tan x$	$\ln|\sec x| + c$
$\cot x$	$\ln|\sin x| + c$

Edexcel

Integration (+ constant)

$f(x)$	$\int f(x)\,dx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln|\sec kx|$
$\cot kx$	$\frac{1}{k}\ln|\sin kx|$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln|\operatorname{cosec} kx + \cot kx|,\quad \frac{1}{k}\ln|\tan(\frac{1}{2}kx)|$
$\sec kx$	$\frac{1}{k}\ln|\sec kx + \tan kx|,\quad \frac{1}{k}\ln|\tan(\frac{1}{2}kx + \frac{1}{4}\pi)|$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

OCR A & B

Integration

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$

Learn: Trigonometric Ratios & Formulae

SOH

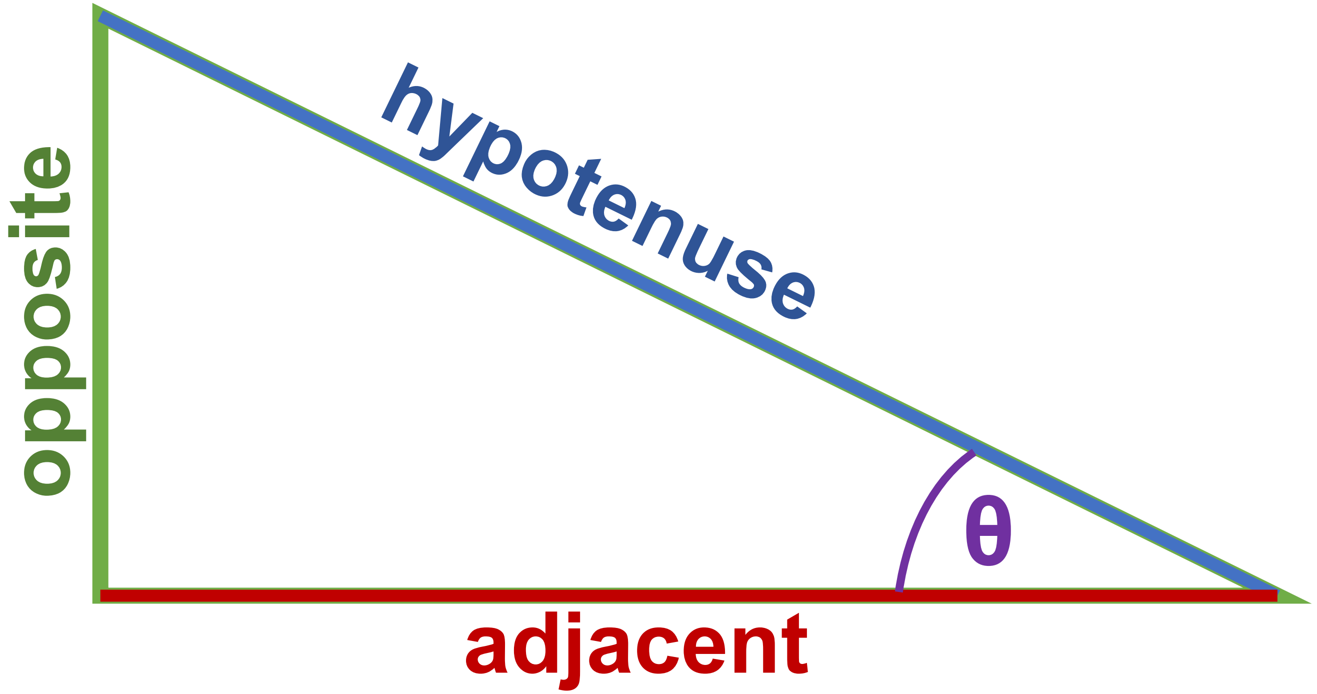
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

CAH

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

TOA

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Mechanics

- Make **units consistent**.
- **Draw** a **diagram** and use it to **figure** out the **applicable method** e.g.
  - Constant acceleration **SUVAT**.
  - Constant velocity **Balancing forces**.
  - Non-linear velocity **Integration** and **differentiation**.

Large Data Set in Statistics

Questions have been chosen to give an **advantage** to those who are **familiar** with the large data set. Learn:

- **Terms** and **abbreviations**
- **Uncommon units** (knots, oktas, cubic cm, physicians per 1000 population etc) that may need converting.
- **Limitations** on data e.g. dates, times, locations.
- **Omitted or N/A** entries should not count towards totals.

Notes, available online for each large data set, highlight **important features** that may be **useful to learn** for the exam.